

Roll No _____ (To be filled in by the candidate)

(Academic Sessions 2020 – 2022 to 2023 – 2025)

MATHEMATICS

224-1st Annual-(INTER PART - I)

Time Allowed : 2.30 hours

PAPER – I (Essay Type)

GROUP – II

Maximum Marks : 80

SECTION – I

LHR-2-24

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2. Write short answers to any EIGHT (8) questions :

(i) Show that $z^2 \bar{z}^2$ is a real number.

(ii) Find the modulus of $1 - i\sqrt{3}$

(iii) Simplify by justifying each step $\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}$

(iv) Check the closure property w.r.t. addition and multiplication for the set $\{0, -1\}$

(v) Determine whether the statement $p \wedge \sim p$ is tautology or not.

(vi) Define semi-group.

(vii) If $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, find $A(\bar{A})'$

(viii) Define reduced echelon form of a matrix, with example.

(ix) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$, verify that $(A^{-1})' = (A')^{-1}$

(x) Discuss nature of roots of $9x^2 - 12x + 4 = 0$

(xi) Solve the equations $x^2 + y^2 = 25$, $2x^2 + 3y^2 = 6$

(xii) Find the condition that one root of $x^2 + px + q = 0$ is square of other.

3. Write short answers to any EIGHT (8) questions :

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(i) Define proper rational fraction.

(ii) For the identity $\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{A}{x-1} + \frac{B}{2x-1} + \frac{C}{3x-1}$ calculate the value of A.

(iii) Find the next two terms of 1, 3, 7, 15, 31, ----

(iv) How many terms are there in the A.P. in which $a_1 = 11$, $a_n = 68$, $d = 3$

(v) Find three A.Ms between $\sqrt{2}$ and $3\sqrt{2}$.

(vi) Find the 12th term of $1 + i$, $2i$, $-2 + 2i$, ----

(vii) Show that ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$

(viii) Evaluate ${}^{12}C_3$

(ix) What is sample space and events?

(x) State principle of mathematical induction.

(xi) Calculate $(9.98)^4$ by means of binomial theorem.

(xii) Prove that $n! > 2^n - 1$ for $n = 4, 5$

4. Write short answers to any NINE (9) questions :

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(i) What is length of an arc intercepted on a circle of radius 14 cm by the arms of a central angle 45° ?

(ii) Convert $54^\circ 45'$ into radians.

(Turn Over)

(2)

4. (iii) If α, β, γ are angles of triangle ABC then prove that $\cos\left(\frac{\alpha + \beta}{2}\right) = \sin\frac{\gamma}{2}$
- (iv) Find the value of $\cos\frac{\pi}{12}$
- (v) Express $\sin(x + 30^\circ) + \sin(x - 30^\circ)$ as a product.
- (vi) Define periodic function and period of trigonometric function.
- (vii) Find period of $\cos\frac{x}{6}$
- (viii) Draw the graph of $y = \sin x$ from 0 to π .
- (ix) State law of sines.
- (x) If sides of triangle are 16, 20, 23, find its greatest angle.
- (xi) Show that $r_1 = s \tan\frac{\alpha}{2}$
- (xii) Find value of $\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)$
- (xiii) Show that $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Solve the system of equations by Cramer's rule : 5
- $$\begin{aligned} 2x + 2y + z &= 3 \\ 3x - 2y - 2z &= 1 \\ 5x + y - 3z &= 2 \end{aligned}$$
- (b) If α, β roots of $x^2 - 3x + 5 = 0$ form the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$ 5
6. (a) Resolve $\frac{x^4}{1-x^4}$ into partial fractions 5
- (b) The sum of an infinite geo-metric series is 9 and the sum of the squares of its terms is $\frac{81}{5}$. Find the series. 5
7. (a) Find the values of n and r when ${}^{n-1}C_{r-1} : {}^nC_r : {}^{n+1}C_{r+1} = 3:6:11$ 5
- (b) If x is so small that its cube and higher powers can be neglected, then show that : $\sqrt{\frac{1-x}{1+x}} \approx 1 - x + \frac{x^2}{2}$ 5
8. (a) Reduce $\cos^4 \theta$ to an expression involving only function of multiples of θ , raised to the first power. 5
- (b) Prove that $r_3 = 4R \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \sin\frac{\gamma}{2}$ 5
9. (a) Show that the area of a sector of a circular region of radius r is $\frac{1}{2}r^2\theta$, where θ is the circular measure of the central angle of the sector. 5
- (b) Prove that $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3 = \frac{\pi}{4}$ 5