Roll No ______________________________ ( To be filled in by the candidate)

**( Academic Sessions 2020 – 2022 to 2023 – 2025 )**

MATHEMATICS 224-1st Annual-(INTER PART – I) Time Allowed : 2.30 hours

PAPER – I ( Essay Type ) GROUP – I Maximum Marks : 80

**SECTION – I**

LHR-1-24

**2. Write short answers to any EIGHT (8) questions :** **16**

(i) Write the symmetric property and transitive property of equality of the real numbers.

(ii) Show that $z\bar{z} = |z|^2 \ \forall z \in C$

(iii) Find out real and imaginary parts of $(\sqrt{3}+i)^3$

(iv) Find the modulus of $1 - i\sqrt{3}$

(v) Construct truth table for $(p \wedge \sim p) \rightarrow q$

(vi) If $a$, b are elements of a group G, then show that $(ab)^{-1} = b^{-1}a^{-1}$

(vii) If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, find the values of $a$ and b.

(viii) If A and B are square matrices of the same order, then explain why in general $(A-B)^2 \neq A^2 - 2AB + B^2$.

(ix) Define skew-hermitain matrix.

(x) Evaluate $\omega^{28} + \omega^{29} + 1$

(xi) When $x^4 + 2x^3 + kx^2 + 3$ is divided by $x-2$, the remainder is 1. Find the value of $k$.

(xii) If $\alpha, \beta$ are the roots of $x^2 - px - p - c = 0$, prove that $(1+\alpha)(1+\beta) = 1 - c$



**3. Write short answers to any EIGHT (8) questions :** **16**

(i) Define partial fractions.

(ii) If $\dfrac{7x+25}{(x+3)(x+4)} = \dfrac{4}{x+3} + \dfrac{B}{x+4}$, then find B.

(iii) Find the number of terms in A.P if $a_1 = 3$ ; $d = 7$ and $a_n = 59$

(iv) If $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in G.P., show that common ratio is $\pm\sqrt{\dfrac{a}{c}}$

(v) Find the sum of $\dfrac{9}{4} + \dfrac{3}{2} + 1 + \dfrac{2}{3} + \text{------} \infty$

(vi) If 5 is H.M. between 2 and b, then find b.

(vii) Write $\dfrac{(n+1)(n)(n-1)}{3.2.1}$ in factorial form.

(viii) Prove that ${}^{n}P_r = n \, . \, {}^{n-1}P_{r-1}$

(ix) Determine probability of getting 2 heads in two successive tosses of balanced coin.

(x) Show that $8.10^n - 2$ is divisible by 6 for n = 1 and n = 2

(xi) Find the $6^{th}$ term in the expansion of $\left(x^2 - \dfrac{3}{2x}\right)^{10}$

(xii) Using binomial theorem, find value of $\sqrt[3]{65}$ correct to three places of decimal.

(Turn Over)

**4. Write short answers to any NINE (9) questions :** **18**

(i) Verify $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ for $\theta = 45°$

(ii) Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$

(iii) If $\alpha, \beta$ and $\gamma$ are the angles of triangle ABC then prove that $\tan(\alpha+\beta) - \tan\gamma = 0$

(iv) Express as product $\cos 6\theta + \cos 3\theta$

(v) Prove that $1 + \tan\alpha\tan 2\alpha = \sec 2\alpha$

(vi) Prove that period of cosine is $2\pi$

(vii) Find the period of $\operatorname{cosec} 10x$

(viii) Draw the graph of the function $y = \cos x, \ x t\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

(ix) Write formula for $\cos\dfrac{\alpha}{2}$ and $\cos\dfrac{\gamma}{2}$

(x) Measure of two sides of a triangle are in the ratio 3 : 2 and angle including these sides is 57°. Find the remaining two angles.

(xi) Define circum centre.

(xii) Without using calculator / table, show that $2\cos^{-1}\dfrac{4}{5} = \sin^{-1}\dfrac{24}{25}$

(xiii) Solve the trigonometric equation $\operatorname{cosec}^2\theta = \dfrac{4}{3}$



## SECTION – II

**Note : Attempt any THREE questions.**

5. (a) Show that $\begin{vmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & b & c+\lambda \end{vmatrix} = \lambda^2(a+b+c+\lambda)$ 5

(b) If the roots of the equation $x^2 - px + q = 0$ differ by unity, prove that $p^2 = 4q + 1$ 5

6. (a) Resolve $\dfrac{1}{(x-3)^2(x+1)}$ into partial fractions 5

(b) Find n so that $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ may be the A.M. between $a$ and b 5

7. (a) Two dice are thrown. $E_1$ is the event that the sum of their dots is an odd numbers and $E_2$ is the event that 1 is the dot on the top of the first die. Show that $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$ 5

(b) If $y = \dfrac{1}{3} + \dfrac{1.3}{2!}\left(\dfrac{1}{3}\right)^2 + \dfrac{1.3.5}{3!}\left(\dfrac{1}{3}\right)^3 + \text{-----}$ prove that $y^2 + 2y - 2 = 0$ 5

8. (a) Reduce $\sin^4\theta$ to an expression involving only function of multiple of $\theta$, raised to the first power. 5

(b) Prove that $\Delta = r^2 \cot\dfrac{\alpha}{2}\cot\dfrac{\beta}{2}\cot\dfrac{\gamma}{2}$ 5

9. (a) Find the values of all the trigonometric functions of the angle $-675°$. 5

(b) Prove that $\sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{7}{25} = \cos^{-1}\dfrac{253}{325}$ 5