FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2021
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let Ψ be a homomorphism of group G into group $\tilde{G}$ with kernel K, prove that K is a normal subgroup of G. (10)

(b) Prove that if H and K are two subgroups of a group G, then HK is a subgroup of G if and only if HK=KH. (10) **(20)**

Q. 2. **(a)** Find elements of the cyclic group generated by the permutation. (10)

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 2 & 6 & 1 \end{pmatrix}$$

(b) Verify that the polynomials $2-x^2$, x^3-x, $2-3x^2$ and $3-x^3$ form a basis for the set $P_3(x)$; the set of all polynomials of degree three. Also express the vectors $1+x^2$ and $x+x^3$ as a linear combination of these basis vectors. (10) **(20)**

Q. 3. **(a)** Let V be the real vector space of all function from R to R. Show that $\{\cos^2 x, \sin^2 x, \cos 2x\}$ is linearly dependent while $\{\cos x, \sin x, \cosh x, \sinh x\}$ are linearly independent. (10)

(b) Solve the system of linear equations: (10) **(20)**

$$\begin{aligned} x_1 - 2x_2 - 7x_3 + 7x_4 &= 5 \\ -x_1 + 2x_2 + 8x_3 - 5x_4 &= -7 \\ 3x_1 - 4x_2 - 17x_3 + 13x_4 &= 14 \\ 2x_1 - 2x_2 + 11x_3 + 8x_4 &= 7 \end{aligned}$$

SECTION-B

Q. 4. **(a)** If $f(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$. (10)

Show that $\frac{\partial^2 f}{\partial y \partial x}(x,y) = \left(\frac{x^2 - y^2}{x^2 + y^2}\right)$

(b) Evaluate $\int_0^6 f(x)dx$ where $f(x) = \begin{cases} x^2 \text{ when } x < 2 \\ 3x - 2 \text{ when } x > 2 \end{cases}$ (10) **(20)**

Q. 5. **(a)** Let $I_n = \int_0^\infty x^n e^{-x} dx$ where n is an integer. Prove that (10)

$I_n = n\, I_{n-1}$ Hence show that $I_n = n!$

(b) **i.** Write $r = \dfrac{8}{2-\cos\theta}$ in rectangular coordinates. (10) **(20)**

ii. Write $x^4 + 2x^2y^2 + y^4 - 6x^2y + 2y^3 = 0$ in polar coordinates.

Q. 6. **(a)** Evaluate $\iint_D dydx$ and $\iint_D dxdy$ where D is the region bounded by the y-axis, the lines x=2 and the curve e^x. (10)

(b) Investigate the curve $y = \dfrac{x^3 - x}{3x^2 + 1}$ for points of inflexion. (10) **(20)**

SECTION-C

Q. 7. **(a)** Sum the series $1 + \frac{1}{2}\cos\theta + \frac{1.3}{2.4}\cos 2\theta + \frac{1.3.5}{2.4.6}\cos 3\theta + \ldots$ (10)

(b) Prove that $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$ (10) **(20)**

Q. 8. **(a)** Construct the analytic function f whose real part is $U = x^3 - 3xy^2 + 3x + 1$ (10)

(b) Evaluate $\int_C \dfrac{dz}{z^2 + 2z + 2}$ Where C is a square with corners (0,0),(-2,0),(-2,-2) and (0,-2). (10) **(20)**

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