FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2020
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. (a) Let G and G' be two groups and $f: G \to G'$ be a homomorphism then prove the following: (10)

(i) $f(e) = e'$ where e and e' are the identities of G and G' respectively

(ii) $f(a^{-1}) = [f(\text{a})]^{-1}, \forall a \in G$

(b) Prove that every homomorphic image of a group is isomorphic to some quotient group. (10) **(20)**

Q. 2. (a) A ring R is without zero divisor if and only if the cancellation law hold. (10)

(b) Prove that arbitrary intersection of subrings is a subring. (10) **(20)**

Q. 3. (a) Let $\text{T} : \text{R}^3 \longrightarrow \text{R}^3$ be the linear transformation defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3)$. Find a basis and dimension of Range of T. (10)

(b) Prove that every finitely generated vector space has a basis. (10) **(20)**

SECTION-B

Q. 4. (a) Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the open intervals on which f is increasing and on which f is decreasing. (10)

(b) Find the horizontal and vertical asymptotes of the graph of $f(x) = -\dfrac{8}{x^2 - 4}$ (10) **(20)**

Q. 5. (a) Calculate $\displaystyle\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$. (10)

(b) Find $\dfrac{\partial w}{\partial x}$ at the point (*x*, *y*, *z*) = (2, -1, 1) if $w = x^2 + y^2 + z^2$, $z^3 - xy + yz + y^3 = 1$ and x and y are the independent variables. (10) **(20)**

Q. 6. (a) Determine the focus, vertex and directrix of the parabola $\text{x}^2 + 6\text{x} - 8\text{y} + 17 = 0$ (10)

(b) Find polar coordinates of the point p whose rectangular coordinates are $\left(3\sqrt{2}, -3\sqrt{2}\right)$ (10) **(20)**

SECTION-C

Q. 7. **(a)** Show that $\left(\cos\theta + i\sin\theta\right)^n = \cos(n\theta) + i\sin(n\theta)$ for all integers *n*. (10)

(b) Find the *n*, *n*th roots of unity. (10) **(20)**

Q. 8. **(a)** Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$. Where, if anywhere, does the series converge to $\frac{1}{x}$? (10)

(b) Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, ($p$ a real constant) converges if $p > 1$, and diverges if $P < 1$ (10) **(20)**

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