UNIVERSITY OF THE PUNJAB

First Semester 2015

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-I
Course Code: MATH-131 /

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

	Short Questions	
Q.02	(a) Solve $\frac{dy}{dx}+3y=3x^2e^{-3x}$. (b) Express the equation $xy=a$ in polar coordinates (c) Evaluate $\int_{\sqrt{2}}^{2}\frac{\sec^2(\sec^{-1}x)dx}{x\sqrt{x^2-1}}$ (d) Find $\frac{d^2y}{dx^2}$ if $y=x\sin x-3\cos x$ (e) State Roll's theorem. (f) Evaluate the integral $\int_{-1}^{5}\|x-3\|dx$ (g) Find absolute extreme values of $f(x)=x^2,\ -2\le x\le 1$ (h) Use L'hopital rule to evaluate $\lim_{x\to a}(x-a)\csc(\frac{x\pi}{a})$ (i) Evaluate $\int e^x \sin x dx$ (j) Find the curve $y=f(x)$ in the xy-plane that passes through the point (9, 4) and whose slope at each point is $3\sqrt{x}$.	[20]
	Long Questions	
Q.03	State Mean Value Theorem and for what values of a, m and b does the function $f(x)=\begin{cases}3 & x=0\\ -x^2+3x+a & 0<x<1\\ mx+b & 1\le x\le 2\end{cases}$ satisfy the hypothesis of the Mean Value Theorem on the interval $[0,2]$?	[10]
Q.04	Find the critical points of $f(x)=2x^3-9x^2+12x$. Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.	[10]
Q.05	a) Solve the initial value problem $(2xy-3)dx+(x^2+4y)dy=0,\quad y(1)=2,$ b) Evaluate $\int\frac{1}{x^2-4x+8}dx$	[10]

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2015
Examination: B.S. 4 Years Programme

PAPER: Calculus (IT)-I
Course Code: MATH-131 / [illegible]

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Objective Type

Q. 01	**Chose the best answer.**	**[10]**

i) If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has its .....

a) maximum value b) minimum value c) both a & b d) none of these

ii) If $f(x) = \sin x$ and $g(x) = \frac{1}{\tan x}$, then $fog(x) = $ ....

a) $\cot(\sin x)$ b) $\sin(\tan x)$ c) $\sin(\cot x)$ d) $\cos(\tan x)$

iii) $\lim\limits_{x \to 1} \frac{x^3 - 1}{x - 1} = $ ----------------------

a) ∞ b) 1 c) 0 d) 3

iv) The range of $f(x) = 2 + \sqrt{x-1}$ is

a) $x \geq 2$ b) $x \geq 1$ c) $x > 1$ d) $x \geq -2$

v) The equations $x = at^2$, $y = 2at$ represent

a) an ellipse b) a parabola c) a hyperbola d) a circle

vi) $3\int_{-\pi}^{\pi} \sin x dx = $

a) 0 b) 1 c) 3 d) -3

vii) $\frac{d}{dx}\left(\cot^{-1} u\right) = $

a) $\frac{du/dx}{1+u^2}$ b) $-\frac{du/dx}{1+u^2}$ c) $\frac{du/dx}{\sqrt{1-u^2}}$ d) $-\frac{du/dx}{\sqrt{1-u^2}}$

viii) The function $f(x) = -3x^2$ is maximum at $x = $ ______________

a) 0 b) 1 c) 2 d) 3

ix) $\frac{d^{999}y}{dx^{999}}(\cos x) = $ ________

a) $\sin x$ b) $-\sin x$ c) $\cos x$ d) $-\cos x$

x) $f(x) = \sin x$ is an increasing function for the domain $(-\pi, \pi)$ in the interval

(a) $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (b) $[-\frac{\pi}{2}, \pi]$ (c) $[\frac{\pi}{2}, \pi]$ (d) $[-\pi, \frac{\pi}{2}]$

UNIVERSITY OF THE PUNJAB

First Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-I
Course Code: MATH-101/

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Short Questions (2x10=20)

Q2.

i. Solve the inequality $|x-5| < 9$.

ii. Evaluate $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$.

iii. If $f(x) = x^2 - 3x + 2$ on $[1,2]$. Discuss the validity of Rolle's Theorem, and find c.

iv. Simplify $(-\sqrt{3} + i)^2$.

v. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}$ find dy/dx.

vi. Evaluate $\int_0^1 x \ln x \, dx$.

vii. Evaluate $\int_{-2}^{2} |x| dx$.

viii. Evaluate $\int \frac{x^2-1}{x^2+1} dx$.

ix. Evaluate $\int \tan^3 x \sec^3 x \, dx$

x. Find a reduction formula $\int \sin^n x \, dx$.

Subjective Questions (6x5=30)

Q3. If

$$f(x) = \begin{cases} x \,, & \text{if } 0 \le x \le 1 \\ 2x-1, & \text{if } 1 < x \le 2 \end{cases}$$

Discuss the continuity and differentiability of f(x) at x = 1.

Q4. Show that $\frac{d^n}{dx^n}\left(\frac{\ln x}{x}\right) = (-1)^n \frac{n!}{x^{n+1}}\left[\ln x - 1 - \frac{1}{2} - \frac{1}{3} - \cdots\cdots\cdots - \frac{1}{n}\right]$

Q5. Evaluate $\int \frac{dx}{3\sin x + 4\cos x}$.

Q6. Use M.V.T. to show that $|\sin x - \sin y| \le |x - y|$ for any real numbers x and y.

Q7. Evaluate $\int_1^{10} \frac{dx}{(x-2)^{2/3}}$.

Q8. Show that $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Mathematics A-1
Course Code: MATH-101

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Objective

Multiple choice questions.

Q1. Encircle the correct answer. **(1x10=10)**

i) If $\frac{2x}{x+2} \geq \frac{x}{x-2}$, then x = -2,2

a) boundary numbers
b) free boundary numbers
c) represents -2 < x < 2
d) solution of given inequality

ii) $\lim_{x \to 0} \frac{sin2x}{sin3x} =$

a) 0
b) 2/3
c) 3/2
d) does not exist

iii) If $z = -\sqrt{3} + i$, then Arg(z) is

a) $-\frac{\pi}{6}$
b) $\frac{\pi}{6}$
c) $\frac{5\pi}{6}$
d) $\frac{3\pi}{2}$

iv) If $f(x) = Sin^2x$ satisfy all the conditions of Rolle's Theorem on $[0,\pi]$, then the value of 'c' is

a) 0 b) $\frac{\pi}{2}$ c) $0, \frac{\pi}{2}$ d) does not exist

v) $\int \frac{dx}{a^2-x^2} =$

a) $\frac{1}{2a} ln\left|\frac{a+x}{a-x}\right|$ b) $\frac{1}{2a} ln\left|\frac{a-x}{a+x}\right|$ c) $\frac{1}{2} ln\left|\frac{a+x}{a-x}\right|$ d) $\frac{1}{2} ln\left|\frac{a-x}{a+x}\right|$

vi) Maclaurin's series for f(x) = *ln*(1-x) is

a) $x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$
b) $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$
c) $-x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots$
d) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

vii) $\int_0^{\pi/2} ln(sinx)dx =$

a) $\frac{\pi}{2} ln2$ b) $\frac{\pi}{2} ln\left(\frac{1}{2}\right)$ c) $\frac{\pi}{2}$ d) $ln2$

viii) $\int_{-1}^{\infty} \frac{1}{x^2} dx =$

a) 1 b) -1 c) 0 d) ∞

ix) $\int_{-1}^{8} \frac{1}{x^{1/3}} dx =$

a) -3/2 b) 3/2 c) -9/2 d) 9/2

x) $\lim_{x \to 0} \frac{e^x - e^{-x}}{sinx} =$

a) 2 b) -2 c) 0 d) does not exist

UNIVERSITY OF THE PUNJAB

First Semester 2015

Examination: B.S. 4 Years Programme

Roll No. ……………………

PAPER: Mathematics B-I [Vectors & Mechanics (1)]
Course Code: MATH-102

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Short Questions (2x10=20)

Q2.

i. Prove that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})=\vec{0}$, where $\vec{a},\vec{b}$ and $\vec{c}$ are non-zero vectors.

ii. If $\vec{a}$ and $\vec{b}$ are unit vectorsand θ is the angle between tham, then show that $Sin\frac{\theta}{2}=\frac{1}{2}|\vec{b}-\vec{a}|$.

iii. Show that div $(r^n\,\vec{r})=(n+3)r^n$.

iv. Find the first and second derivative of $\vec{r}\times\left(\frac{d\vec{r}}{dt}\times\frac{d^2\vec{r}}{dt^2}\right)$.

v. State and prove varigon's theorem.

vi. Define moment of a force about a fixed point.

vii. Forces act along the sides BC, CA, and AB of a $\Delta\,ABC$. Show that they are equivalent to a couple only, if the forces are proportional to their sides.

viii. Describe laws of friction.

ix. Find the least force to drag down the particle on a rough inclined plane.

x. State the principle of virtual work for a single particle.

Subjective Questions (6x5=30)

Q3. Prove that $\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}$ is equally inclined with $\vec{a}$ and $\vec{b}$.

Q4. If $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$, then prove that $\nabla\, r^n=nr^{n-2}\vec{r}$.

Q5. State and prove (λ,u) theorem.

Q6. If forces $l\overrightarrow{AB}$, $m\overrightarrow{BC}$, $l\overrightarrow{CD}$ and $m\overrightarrow{DA}$ acting along the sides of a quadrilateral are equivalent to a couple. Show that either $l=m$ or ABCD is a parallelogram.

Q7. Find the least force to drag a particle up on a rough inclined plane.

Q8. A uniform rod of length 2a rest in equilibrium against a smooth vertical wall and upon a smooth peg at a distance 'b' from the wall. Show that in the position of equilibrium the rod is inclined to the wall at an angle of $Sin^{-1}\left(\frac{b}{a}\right)^{1/3}$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-I [Vectors & Mechanics (1)] **TIME ALLOWED: 30 mins.**
Course Code: MATH-102 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Multiple choice questions. **(1x10=10)**

Q1. Encircle the correct answer.

i) If $\vec{a}.\vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$ then:

a) $\vec{a}$ is perpendicular to $\vec{b}$ b) $\vec{a}$ and $\vec{b}$ are collinear
c) $\vec{a}$ or $\vec{b}$ is a null vector d) $\vec{a} = \vec{0}, \vec{b} = \vec{0}$

ii) If $\vec{a}.(\vec{b} \times \vec{c}) = 0$, then $\vec{a}, \vec{b}$ and $\vec{c}$ are:

a) Coplanar b) collinear c) mutually perpendicular d) null vectors

iii) If $\vec{A} = 5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}$, then $\frac{d}{at}(\vec{A}.\vec{A}) =$:

a) $100t^3 + 2t + 6t^5$ b) $10t^3 + 2t^2 + 5t^5$
c) $100t^4 - 2t^2 - 6t^4$ d) $-100t^3 - 2t - 6t^5$

iv) $\nabla.(r^3\vec{r}) =$:

a) r^3 b) $6r^3$ c) 0 d) $3r^3$

v) The state of rest of body relative to other bodies is called:

a) equilibrium b) non-equilibrium c) force d) couple

vi) Let the forces $\vec{F_1}$ and $\vec{F_2}$ act at '0'. Then their resultant by the parallelogram of forces is $\vec{R} =$:

a) $\vec{0}$ b) $\vec{F_1} + \vec{F_2}$ c) $\vec{F_2} - \vec{F_1}$ d) $\frac{F_1+F_2\cos\alpha}{\cos\theta}$

vii) A set $(\vec{F}, -\vec{F},)$ of two parallel opposite force with same magnitude acting on a rigid body form a:

a) Moment b) couple
c) magnitude of a couple d) magnitude of a moment

viii) The unit of co-efficient of friction is:

a) Newton b) dyne c) horse power d) none of these

ix) The maximum amount of friction which can be called into play is called:

a) static friction b) kinetic friction c) limiting function d) none of these

x) If zero vitual work is done by constraint forces in any virtual displacement of system, then constraints are called:

a) internal forces b) external forces c) workless d) applied forces

UNIVERSITY OF THE PUNJAB

First Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ……………………

PAPER: Elementary Mathematics-I (Algebra)
Course Code: MATH-111

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SHORT QUESTIONS

Q.2 Solve the following Short Questions: **(2 × 10 = 20)**

(i) Show that $z^2+(\bar{z})^2$ is a real number for all $z\in C$

(ii) If $A=\begin{bmatrix}\cos^2\theta+1 & \cos\theta\sin\theta\\ -\cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $B=\begin{bmatrix}\sin^2\theta & -\cos\theta\sin\theta\\ \cos\theta\sin\theta & \cos^2\theta+1\end{bmatrix}$, then show that $A+B$ is a scalar matrix $2I_2$

(iii) Evaluate $\left(\frac{-1+\sqrt{-3}}{2}\right)^7+\left(\frac{-1-\sqrt{-3}}{2}\right)^7$

(iv) Find the A.M of $1-x+x^2$ and $1+x+x^2$

(v) Which term of the sequence $-6,-2,2,\ldots$ is 70?

(vi) Find the term involving x^3 in the expansion of $(x+2)^5$

(vii) Find the nth term of the H.P. $\frac{1}{9},\frac{1}{12},\frac{1}{15},\ldots$.

(viii) Find r when $\theta=\frac{\pi}{7}$ radians, $l=1$ cm

(ix) Prove that $\sin(\frac{\pi}{4}+\alpha)=\frac{1}{\sqrt{2}}(\sin\alpha+\cos\alpha)$

(x) Find the area of a sector with central angle of 0.25 radian in a circular region whose radius is 2 m

Subjective Questions **(6 × 5 = 30)**

Q.3 Evaluate $\begin{vmatrix} b & -1 & a\\ a & b & 0\\ 1 & a & b\end{vmatrix}$

Q.4 Solve the system of linear equations by Cramer's rule

$2x+y+z=1,\quad 3x+y-5z=8,\quad 4x-y+z=5$

Q.5 Show that the roots of the equation $x^2+(mx+c)^2=a^2$ will be equal if $c^2=a^2(1+m^2)$

Q.6 If the 5^{th} term of A.P is 16 and 20^{th} term of A.P is 46. Find the 12^{th} term of A.P

Q.7 Prove that $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{2}{1-2\sin^2\theta}$

Q.8 If $x=a\sin\theta-b\cos\theta$ and $y=a\cos\theta+b\sin\theta$, then show that $x^2+y^2=a^2+b^2$

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-I (Algebra)
Course Code: MATH-111

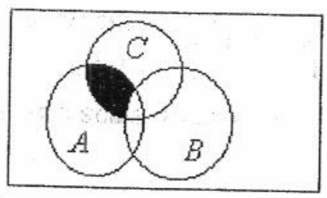
TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Multiple Choice Questions

Q.1 Some possible answers are given for the following questions. Tick (√) the correct answer.

(i) The shaded region represents



(a) $(A\cup B)\cap C$ (b) $(A\cap C)-B$ (c) $(B\cap C)-A$ (d) $(A\cap B)-C$

(ii) The order of the matrix $[2 \quad 5 \quad 7]$ is

(a) 3×3 (b) 1×1 (c) 3×1 (d) 1×3

(iii) If $\begin{vmatrix} -1 & 3 \\ x & 1 \end{vmatrix} = 0$, then $x =$ ________

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

(iv) The Product of all four fourth roots of unity is

(a) 1 (b) -1 (c) 4 (d) 16

(v) If $a_{n-2} = 3n-11$, then 5th term is:

(a) 4 (b) 7 (c) 10 (d) 13

(vi) If $a=3, r=2$, then the nth term of the G.P. is

(a) 2.3^{n-1} (b) 3.2^{n} (c) 3.2^{n+1} (d) 3.2^{n-1}

(vii) The number of terms in the expansion of $(2a+b)^{10}$ is:

(a) 10 (b) 11 (c) 12 (d) 13

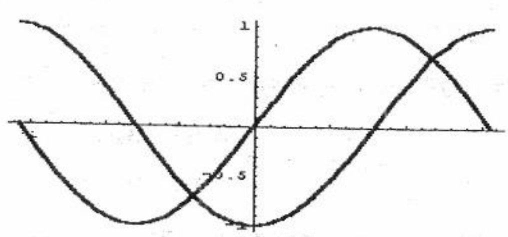
(viii) The expansion of $(1-3x)^{\frac{2}{3}}$ is valid if

(a) $|x|<\frac{1}{3}$ (b) $|x|<\frac{1}{2}$ (c) $|x|<\frac{2}{3}$ (d) $|x|<1$

(ix) $\cos^2\theta+\sin^2\theta=$

(a) -2 (b) -1 (c) 0 (d) 1

(x) Two trigonometric functions are drawn taking same scale from $-\pi$ to π in the following graph, it represents



(a) $\cos x$ and $\sec x$ (b) $\sin x$ and $\csc x$
(c) $\cos x$ and $\sin x$ (d) $-\cos x$ and $\sin x$

UNIVERSITY OF THE PUNJAB

First Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Business Mathematics
Course Code: MATH-112

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Subjective Type

Q.2 Answer the short questions. **(10x2)**

i) Find inverse of the matrix $\begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix}$

ii) Find the roots of $x^2 + 7x + 12 = 0$.

iii) What is the difference between combinations and permutations?

iv) Find two consecutive odd integers whose sum is 36.

v) Find the 6^{th} term of progression 2, 6, 18, ...?

vi) In how many ways the letters of the word ASSASSINATION can be arranged?

vii) Find the number of years for Rs. 7450 to earn 1788 simple interest at 12% per annum.

viii) The price of 15 books is Rs.900, find the cost of such 60 books?

ix) Solve for x and y if $x - 2y = -3, 2x - y = 1$.

x) Find the number of terms in an A.P in which $a_n = 30, d = 2, a_1 = 2$.

Long Questions

Q.3 What is the compound interest on Rs.1000 for 4 years at 5% compounded annually? (6)

Q. 4 At what rate Rs. 5000 double itself in 5 years. (6)

Q.5 Using logarithms, solve $\frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$. (6)

Q.6 Find four numbers in A.P whose sum is 32 and sum of whose squares is 276. (6)

Q.7 Solve the system of equations (6)

$$x - 2y + z = -1$$
$$y - Z = 1$$
$$3x + y - 2z = 4$$

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2015
Examination: B.S. 4 Years Programme

PAPER: Business Mathematics
Course Code: MATH-112

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.
Objective Type

Q.1 Tick on the correct option

i) The regular fixed-periodic sequence of payments charged with compound interest is called

a) Annuity b) Amount c) Simple discount d) Simple interest

ii) The roots of quadratic equation $x^2 - 7x + 6 = 0$ are

a) -1, -6 b) -1, 6 c) 1, 6 d) 1, -6.

iii) The value of ${}^6P_1$ is

a) 18 b) 12 c) 6 d) 0

iv) The order of matrix $[1 \quad 2 \quad 8 \quad -3]$ is

a) 4×1 b) 1×4 c) 4×4 d) 4×3

v) The sum of infinite geometric series can be found if

a) $|r| < 1$ b) $|r| > 1$ c) $|r| \le 1$ d) $|r| = 0$

vi) If $A = \begin{bmatrix} 4 & -1 & 0 \\ 7 & 3 & 1 \\ 2 & x & 2 \end{bmatrix}$ and $|A| = 4$, then the value of x is

a) 8 b) -8 c) 4 d) -4

vii) Mr. X bought a T.V set for Rs. 187.50 and sold at Rs. 250. The profit percentage he made is

a) 30% b) 33.3% c) 35% d) 37.5%

viii) The exponential form of $x = \log_a y$ is

a) $a = y^x$ b) $y = a^x$ c) $x = y^a$ d) $y = x^a$

ix) A man spends 96% of his income and save Rs. 525. Then his income is

a) 13000 b) 13 100 c) 13125 d) None of these

x) The general term of Geometric Sequence is

a) $a_n = a_1 r^{n-1}$ b) $a_n = a_1 r^{n+1}$ c) $a_n = a_{n-1} r^{n-1}$ d) $a_n = a_{n-1} r^{n+1}$

UNIVERSITY OF THE PUNJAB

First Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus-I
Course Code: MATH-121

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Note: Attempt all the questions.

Section-II (Subjective)

Q. No. 2 Briefly give the short answers of the following questions. 2 × 10 = 20

i. Find the following limit

$$\lim_{x\to a}(x-a)cosec\,(\frac{\pi x}{a})$$

Using L-Hospital Rule

ii. Define the convergence sequence.

iii. What is an ellipse? Also write the equation for the ellipse?

iv. Define the continuity of a function.

v. Write the Taylor's series?

vi. What do you mean about differentiation? Also write the differentiate the following function

$f(x) = \frac{x}{x-1}$

vii. What are the polar coordinates? Also write the relation of the polar coordinates with the Cartesian coordinates.

viii. Find the distance between the points P_1 (2, 1, 5) and P_2 (-2, 3, 0).

ix. Find the unit vector perpendicular the plane of P(1, -1, 0), Q(2, 1, -1) and R(-1, 1, 2).

x. Evaluate the limit

$$\lim_{x\to 0}\frac{\sin ax}{\sin bx}$$

P.T.O.

Q. No. 3 (a) State and prove the mean value theorem? 6

(b) Write the minimum five properties of the limits. 4

Q. No. 4(a) Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{n!\, x^n}{(2n)!}$ 5

(b) Find the tangent and normal to the curve

$$x^2 - xy + y^2 = 7$$

At the points (-1, 2). 5

Q. No. 5(a) differentiate the following function Y w.r.t. x

$$y(x) = \frac{(x+2)^2}{(x+1)(x^2+3)^3}$$ 4

(b) Find the value of 'c' such that the function

$$f(x) = \begin{cases} \frac{1-\sqrt{x}}{x-1}, & if\ 0 \le x < 1 \\ c, & if\ x = 1 \end{cases}$$

Is continuous, for all $x \in [0, 1]$. 3

(c) find the value of " c " using the mean value theorem 3

Where the function is

$$y(x) = x^2 + 2x - 1 \quad \text{for } [0, 1]$$

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Calculus-I
Course Code: MATH-121

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Note: Attempt all the questions

Section-I (Objective)

Q. No.1. Each question carries the four options but encircle the serial number of one best answer of the question. Cutting, over writing and rewriting will be considered as a wrong. 1 × 10 = 10

I. If a , b and c are real numbers and a < c and c > 0 then

(a) ac < bc (b) ac > ac (c) ac = bc (d) none of these

II. A finite interval will be ...... interval if it contains both end points.

(a) Half open interval (b) half closed interval (c) open interval (d) closed interval

III. Solving the inequality $6x - 8 < x + 7$, the x will be

(a) 2 (b) 3 (c) 1 (d) 0

IV. The fourth derivative of the function $y = x^3 - 3x^2 + 2$ is

(a) 6 (b) 6x-6 (c) zero (d) none of these

V. The polar coordinates are

(a) ρ, r, θ (b) r, θ (c) x, y ,z (d) r, θ, z

VI. If $y = sex\ x\ then$ $\frac{d^2y}{dx^2}$ will be

(a)$sec^3x + \sec(x)\ tan^2x$ (b) $sec^2x + \sec(x)\ tan^2x$

(c))$sec^3x + sec^2(x)\ tan^2x$ (d) none of these

VII. Evaluating the $\lim_{x\to 0} \frac{\sin 2x}{5x}$ will be

(a) 1/5 (b) 2 (c) 2/5 (d) zero

VIII. The focus of the parabola $y^2 = 10\,x$ is

(a) (-5/2, 0) (b) (0, -5/2) (c) (5/2, 0) (d) (10, 0)

IX. The absolute extreme values of $g(t) = 8t - t^2$ on [-2, 1]

(a) -32 (b) $2^{1/2}$ (c) 7 (d) none of these

X. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the equation of

(a) Hyperbola (b) Parabola (c) Ellipse (d) a and b are correct.

UNIVERSITY OF THE PUNJAB

First Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Applied Mathematics
Course Code: MATH-122

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Section II

Note : Attempt all questions showing precise calculations. 5×4=20

1. If x is Poisson random variable with parameter μ=2, find the probabalities for x=0, 1, 2, 3
2. Evaluate $\int_0^2 \frac{dx}{1+x^2}$ by using the trapezoidal rule for n= 4,compare with exact value.
3. Find the root of the equation, $x^3 - x - 11 = 0$, by Secant method, perform three iterations.
4. For two events A and B of a sample space such that A is a subset of B, prove that P(A)≤P(B)
5. For the following data compute regression coefficients b_{xy}, b_{yx} and correlation coefficient r

x	8.2	9.6	7.0	9.4	10.9
y	8.6	9.6	6.9	8.5	11.3

Section III .

Note: Give detailed answers to the following questions. 3×10=30

1. (a) Define Poisson probability distribution function and prove that it is a limiting case of the Binomial distribution function. (6)
   (b) Let x have a binomial distribution with n=4 and p=1/3, find P(x=1), P(x=3/2) (4)

2. (a) Derive the formula for the Secant method and also write its Algorithm. (6)

   (b) Use the rectangular for n=3 to evaluate $\int_0^6 \sqrt{x^2+1}\,dx$ (4)

3. (a) Define the correlation coefficient and prove that it is invariant with respect to origin and scale. (6)

   (b) find Y= a+bX for the given data (4)

X	20	40	60	80	90	100
Y	12	10	8	6	4	2

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Applied Mathematics
Course Code: MATH-122

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Section I

Q1. Choose the best answer for the following statements: Marks: $1\times10=10$

1. If a density function is defined as:

   $f(x) = kx$, $0 \le x \le 2$

   $f(x) = 0$, otherwise , then value of k is

   i) 1 ii) 1/2 iii) 3/2 iv) None

2. Let X have a binomial distribution with n= 4, then P(X=3) is

   i) 32/81 ii) 0 iii) 8/81 iv) None

3. Newton Raphson formula is defined as:

   i) $x_n = x_{n-1} - \dfrac{f(x_{n-1})}{f'(x_{n+1})}$ ii) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

   iii) $x_{n+2} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ iv) None

4. If a function 'f' is defined on [a,b], then to find its root by Bisection method, we must have,

   i) f(a).f(b)=0 ii) f(a).f(b)<0 iii) f(a).f(b)>0 iv) None

5) The Number of roots of the equation $x^3-x+1=0$ is

   i) 1 ii) 2 iii) 3 iv) None

6) A fair coin is tossed three times, the number of elements in the sample space is

   i) 2 ii) 4 iii) 6 iv) 8

7) Median of 4, 18,18,20 is

   i)18 ii) 17 iii) 8 iv) None

8) To use the Simpson 1/3 Rule , n number of sub intervals are used where n is

   i) even ii) odd iii) both even and odd iv) none

9) The rectangular rule is a method to evaluate

   i) nonlinear equations ii) definite integral iii) system of linear equations iv) None

10) To apply the Jacobi iterative method , the diagonal elements of the matrix must be

   i) zero ii) non-zero iii) positive iv) negative.

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Roll No. ....

Second Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Mathematics A -II, [Plane Curves & Analytic Geometry] **TIME ALLOWED: 30 mins.**

Course Code: MATH-103 / **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Multiple Choice Questions

Q. # 1: Encircle the correct answer **(1x10=10)**

(i) The locus of middle points of a system of parallel chords of an ellipse is called:

(a) Diameter (b) eccentricity (c) directrix (d) Major axis

(ii) In rectangular coordinate system $r = a\sin\theta(a>0)$ is:

(a) $x^2+y^2 = ay$ (b) $x^2-y^2 = ay$ (c) $x^2+ay = y^2$ (d) $x^2 = ay+y^2$

(iii) $r = a\,(1+\cos\theta)$ is the equation of a:

(a) Parabola (b) circle (c) cardioid (d) lemniscate

(iv) The asymptotes of the curve $y = \dfrac{2(x-2)}{x^2}$ are:

(a) x=0 (b) y=0(c) x=0 and y=0 (d) x=2 and y=0

(v) If α, β and γ are the direction angles of a line, then

(a) $Cos\alpha^2 + \cos^2\beta + \cos^2\gamma = 2$ (b) $Cos\alpha^2 + \cos^2\beta + Cos^2\gamma = 1$

(c) $Sin^2\alpha + Sin^2\beta + Sin^2\gamma = 1$ (d) $Cos2\alpha + \cos2\beta + \cos2\gamma = 2$

(vi) The distance of the point A(3, -1,2) to the plane 2x+y-z-4=0 is:

(a) $\dfrac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\dfrac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

(vii) A point through which there passes two branches of a curve is called:

(a) A singular point (b) double point (c) origin (d) conjugate

(viii) The radius of curvature =

(a) $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$ (b) $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{dy}{dx}}$ (c) $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{1/2}}{\dfrac{dy}{dx}}$ (d) $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{2}}{\dfrac{d^2y}{dx^2}}$

(ix) For a curve a relation between S and α is called:

(a) intrinsic equation (b) pedal equation

(c) polar equation (d) equation of normal

(x) Angle between the pair of lines $3x^2+7xy+2y^2=0$ is of measure:

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

UNIVERSITY OF THE PUNJAB

Second Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A -II,
[Plane Curves & Analytic Geometry]
Course Code: MATH-103 /

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Q. # 2: **Short Questions** **2x10=20**

(i) Find the equation of the normal to the curve $xy = c^2$ at $\left(ct, \frac{c}{t}\right)$

(ii) Find the pedal equation of the parabola $y^2 = 4a(x+a)$

(iii) Define diameter and conjugate diameters of an ellipse

(iv) Show that the curve with parametric equations $x = a\cos\theta + h, y = b\,Sin\theta + k$ is an ellipse with centre (h,k).

(v) Define cusp.

(vi) Determine the point t, if any common to the straight line $x=1+t,\ y=t, z=-1+t$ and the plane $x+y+z=3$.

(vii) Express $x^2+y^2+2z=6$ in spherical coordinates.

(viii) Find equation of the asymptotes of the curve $r = \frac{a}{\theta}$

(ix) Transform $x^2+y^2-z=9$ into spherical coordinates.

(x) Find the area of the region included within the cardioids $r = a(1-\sin\theta)$.

Subjective Questions **5x6=30**

Q. # 3: Identify and graph the polar equation $r = \frac{4}{1+Cos\theta}$

Q. # 4: A forms has 1000 meters of barbed wire which he is to fence off three sides of a rectangular field, the fourth side being bounded by a straight canal. How can the former enclose the largest field.

Q. # 5: Show that the pedal equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}-\frac{r^2}{a^2b^2}$$

Q. # 6: Find the asymptotes of the curve $x^2y+xy^2+xy+y^2+3x=0$.

Q. # 7: Show that the distance of the point $P\ (3,-4,5)$ from the plane $2x+5y-6z=16$, measured parallel to the line $\frac{x}{2}=\frac{y}{1}=\frac{z}{-2}$ is $\frac{60}{7}$.

Q. # 8: Find the intrinsic equation of the cardioid $r = a(1-Cos\theta)$.

UNIVERSITY OF THE PUNJAB

Roll No. ………………

Second Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-II [Mechanics(II)]
Course Code: MATH-104

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Q. # 1: Multiple choice questions. Encircle the correct answer. **(1x10=10)**

(i) The normal component of velocity of a particle moving in a plane is:

(a) v (b) 0 (c) $\frac{v^2}{\rho}$ (d) $\frac{v}{\rho}$

(ii) The transversal component of acceleration of a particle moving in a plane is:

(a) $2\dot{r}\dot{\theta}+r\ddot{\theta}$ (b) $\ddot{r}-r(\dot{\theta})^2$ (c) $r\dot{\theta}+\ddot{\theta}$ (d) $\ddot{r}-r\ddot{\theta}$

(iii) The most simple motion is called:

(a) Rectilinear (b) S.H.M (c) Projectile motion (d) Circular motion

(iv) If a particle is moving with an acceleration directed towards the mean position, then the motion is said to be:

(a) Rectilinear motion (b) Curvilinear

(c) S.H.M (d) Projectile motion

(v) In case of S.H.M, the maximum velocity is:

(a) $\sqrt{\lambda}\,a$ (b) $\sqrt{\lambda a}$ (c) 0 (d) λa

(vi) The product of frequency and time period is:

(a) 1 (b) 0 (c) λ (d) 2π

(vii) The force $\vec{F}$ is said to be conservative, if :

(a) $Curl\ \vec{F}=\vec{0}$ (b) $\vec{F}=0$ (c) $Curl\ \vec{F}\neq\vec{0}$ (d) $dir\ \vec{F}\neq 0$

(viii) A particle projected with speed v_o and angle of projection $\propto$ have the maximum range as:

(a) $\frac{v_o^2\sin 2\alpha}{g}$ (b) $\frac{v_o^2\cos 2\alpha}{g}$ (c) $\frac{v_o^2}{g}$ (d) $\frac{v_o}{g^2}$

(ix) Parabola of safety is also called:

(a) Fixed parabola (b) Rectangular parabola

(c) Circular parabola (d) Harmonic parabola

(x) The force which is always directed towards or away from a fixed point is called:

(a) Conservative force (b) Central force

(c) Internal force (d) Constraint force

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Second Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-II [Mechanics(II)] **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-104 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q. # 2: Short questions **(2x10=20)**

(i) Write the radial and transversal components of velocity and acceleration of a particle moving in a plane.

(ii) If a particle is moving along the curve $\vec{r} = a\cos t\,\hat{i} + b\sin t\,\hat{j}$. Find the velocity and acceleration of the particle at any time t.

(iii) Find the velocity attained by a particle moving in a straight line at any time 't' if it starts from rest at t = o and subject to an acceleration $t^2 + \sin t + e^t$.

(iv) A stone is let fall freely from a height of 100feet. Find the time that it takes and the velocity that it acquires on reaching the ground.

(v) A cannon has its maximum range R. prove that

(a) The height reached is $\frac{R}{4}$ (b) The time of flight is $\sqrt{\frac{2R}{g}}$

(vi) Define parabola of safety and write its equation.

(vii) A uniform rod AB is 4 feet long and weight 6 lb. the weights attached to it are as follows. 1 lb at A, 2ld at 1 foot from A, 3 lb 2 feet from A, 4 lb at 3 feet from A and 5lb at B. Find the distance from A of the centre of gravity of the system.

(viii) Define apse and apsidal distance.

(ix) Define harmonic oscillator and Damped Harmonic oscillator.

(x) State Kepler's laws of planetary motion.

Subjective questions **(5x6=30**

Q. # 3: The position of a particle moving along an ellipse is $\vec{r} = a\cos t\,\hat{i} + b\sin t\,\hat{j}\,(a > b)$. Find the position of the particle where its velocity has maximum and minimum magnitude.

Q. # 4: A particle is projected vertically upwards with a velocity $\sqrt{2gh}$, another is let fall from a height 'h' at the same time. Find the height of the point where they meet each other.

Q. # 5: The force act on a particle of mass m at time. 't' is given by $\vec{F} = a\cos wt\,\hat{i} + b\,Sin\,wt\,\hat{j}$. If the particle is initially at rest at the origin, find the position and velocity of the particle at any time 't'.

Q. # 6: A shall burst on contact with the ground and pieces from it fly in all directions with all speed upto 80 feet per second. Prove that a man 100 feet away is in danger for $\frac{5}{\sqrt{2}}$ seconds.

Q. # 7: A particle describes the curve $r^n \cos n\theta = a^n$ under a force F to the pole. Show that $F \alpha r^{2n-3}$.

Q. # 8: Calculate the centroid of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ lying in the first quadrant.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Discrete Mathematics
Course Code: MATH-105

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

(OBJECTIVE)

Q. # 1: ✓ or encircle the correct answer. (10)

(i) $\neg p \rightarrow q$ is equivalent to.

(a) $p \rightarrow q$ (b) $\neg p \wedge q$ (c) $\neg p \vee q$ (d) $\neg p \rightarrow \neg q$

(ii) The number of rows in a truth table of a proposition involving three variables are:

(a) 3 (b) 4 (c) 8 (d) 6

(iii) With usual notations $\lceil 1.2 \rceil + \lfloor 3.5 \rfloor =$.

(b) 4 (b) 5 (c) 6 (d) 3

(iv) $If\ f(x)=x^2+\frac{1}{x^2} then\ f\left(\frac{1}{x}\right)=$.

(a) x^4+1 (b) $x^2+\frac{1}{x^2}$ (c) $f(x)$ (d) both b and c

(v) If aRa for all $a \in S$, then relation, , R, is called:

(a) Reflexive (b) Symmetric (c) Transitive (d) None

(vi) ${}^{7}C_4 + {}^{7}C_3 =$

(a) ${}^{7}C_4$ (b) ${}^{8}C_4$ (c) ${}^{14}C_7$ (d) ${}^{49}C_{12}$

(vii) How many two digit numbers when digits can be repeated:

(a) 90 (b) 81 (c) 80 (d) 100

(viii) $If\ f(0)=2$, $f(1)=3$, $f(n+1) = 2\ f(n-1) + f(n)$ **then** $f(2)=$

(a) 4 (b) 7 (c) 6 (d) 5

(ix) ${}^{n}C_{n-r} =$

(a) ${}^{n}C_n$ (b) ${}^{n}C_r$ (c) ${}^{r}C_r$ (d) None

(x) Which of the following is a poset

(a) $(z, \neq)$ (b) (Z, $\nmid$) (c) $(Z, \leq)$ (d) None

Where Z is the set of all integers.

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Second Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics
Course Code: MATH-105

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

(SUBJECTIVE)

Q. # 2: Solve the following "Sort Questions". **(2x10=20)**

(i) Show that $\neg(p \vee q)\, and \neg p \wedge \neg q$ are logically equivalent.

(ii) State the Binomial Theorem.

(iii) With usual notation prove that $C_r^n = C_{n-r}^n$

(iv) Expand $(a+b)^4$.

(v) Write an equivalence relation definie on a set $\{a,b,c,d\}$

(vi) Show that $1+2+3+.......+n=\dfrac{n(n+1)}{2}$.

is true for all positive integers n.

(vii) Prove that $|x+y|\leq|x|+|y|\,;\ \forall x,y\in R$.

(viii) Define Pigeonhole Rule.

(ix) Show that $\dfrac{x^4+1}{x^3+x^2}$ is $O(x)$, $(O\ is\ big-oh)$

(x) How many arrangements of the letters of the word PLACE, each letter use once.

Q. # 3: Solve the following "Long Questions" **(5x6=30)**

(i) If $f(x)=\dfrac{7x-3}{x+4}$ $find\ f^{-1}(x)\,\&\,verify: f\left(f^{-1}(x)\right)=x.$

(ii) Show that $\neg p\rightarrow(p\rightarrow q)$ is a tautology without using truth table.

(iii) Prove that for every positive integer, n,

$1.2+2.3+......+n(n+1)=n(n+1)(n+2)/3.$

(iv) How many permutations of the letters ABCDEFGH contain

(a) strings AFG and (b) Strings BC and CFG.

(v) List all the 3 – permutation of the set $\{a,b,c,d,e\}$.

(vi) For a set $\{a,b,c,d\}$, Discuss

$\{(a,a),(a,b),(b,a),(b,b),(c,c),(c,d),(d,c),(d,a),(a,d),d,d)\}$

For equivalence relation and partially ordered relation.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-I (Algebra)
Course Code: MATH-111

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Objective type

Q.1 Tick on the correct option. (10)

(i) The additive identity of real numbers is

(a) 0 (b) 1 (c) 2 (d) 3

(ii) If $z = -2 - 7i$, then $|\bar{z}| =$

a) $\sqrt{45}$ b) $\sqrt{49}$ c) $\sqrt{53}$ d) $\sqrt{57}$

(iii) Three consecutive integers are such that three times the sum of the larger pair is equal to five times the sum of the smaller pair. The numbers are

(a) 11, 12, 13 (b) 1, 2, 3 (c) 8, 9, 10 (d) 2, 3, 4

(iv) If A is a matrix of order 3×2 then the order of $A^t A =$ is

(a) 3×3 (b) 2×3 (c) 2×2 (d) 3×2

(v) Terminating numbers are also called

a) Irrational numbers b) rational numbers c) integers d) natural numbers

(vi) $1 + \omega + \omega^2 =$

(a) 1 (b) ω (c) ω^2 (d) 0

(vii) The second term of the sequence with general term $\frac{n^2 - 4}{2}$ is

(a) 3 (b) -3 (c) 1 (d) 0

(viii) The second term in the expansion of $(1 - 2x)^{-\frac{1}{2}}$ is:

(a) x (b) $2x$ (c) $3x$ (d) $4x$

(ix) The period of $\sin x$ is:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) 2π

(x) Product of the roots of equation $ax^2 + bx + c = 0$ is

a) $\frac{b}{a}$ b) $-\frac{b}{a}$ c) $\frac{c}{a}$ d) $-\frac{c}{a}$

UNIVERSITY OF THE PUNJAB

Second Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-I (Algebra)
Course Code: MATH-111 / [illegible]

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Q.2 Answer the short questions (10x2=20)

(i) If $z_1 = 3 - i$, evaluate $\text{Re}(-3z_1)$.

(ii) Find $g(x)$ given that $g(x+h) = 7(x+h)^2 + 8(x+h) + 5$.

(iii) Determine x if $\begin{vmatrix} 5 & 2x & 0 \\ 1 & x & 4 \\ -1 & 3 & 1 \end{vmatrix} = -10$.

(iv) For what values of m the equation $3mx^2 = 4(mx-1)$ will have equal roots?

(v) Solve the quadratic equation $3x^2 + 17x - 20 = 0$.

(vi) Find the nth term of the sequence $\left(\frac{5}{4}\right)^2, \left(\frac{9}{4}\right)^2, \left(\frac{13}{4}\right)^2, \ldots$

(vii) Find the expansion of $(x - 5y)^5$.

(vii) Prove that $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{\cot\theta + 1}{\cot\theta - 1}$.

(ix) Find the area of a sector with central angle of 0.25 radian in a circular region whose radius is 2 m.

(x) Bilal wants to cover the floor of wash room measuring 6 ft by 10 ft with square tiles of the same size. Given that he uses only whole tiles, find the largest possible length of the side of each tile.

(P.T.O.)

Long Questions (6x5=30)

Attempt all questions.

Q.3 The result of an examination of 50 students in two subjects is shown below:

Subject	Pass	Fail
Chemistry	37	13
Biology	33	17

If 9 were failed in both subjects. How many were passed in both subjects?

Q.4 Use Cramer's rule to find the solution of system of equations:

$$2x - y + z = 1$$
$$3x + y - 5z = 8$$
$$4x + y + z = 5$$

Q.5 One pipe can fill a pool 1.25 times faster than a second pipe. When both pipes are opened, they fill the pool in five hours. How long would it take to fill the pool if only the slower pipe is used?

Q.6 Using mathematical induction, prove that

$$3 + 7 + 11 + \ldots + (4n - 1) = n(2n + 1)$$

Q.7 If $\sin\alpha\cos\beta = p$ and $\cos\alpha\sin\beta = q$, then find the value of $\sin(\alpha+\beta)\sin(\alpha-\beta)$ in terms of p and q.

Q.8 Prove that $(1 + \tan\alpha - \sec\alpha)(1 + \cot\alpha + \operatorname{cosec}\alpha) = 2$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Calculus-II
Course Code: MATH-123 /

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Objective type

Q.1 Tick on the correct option. [1x10]

i) If $f(x,y)=\ln(x^2+y^2)$ then first order partial derivative $f_x(x,y)$ is

a) $\frac{2}{x^2+y^2}$ b) $\frac{2x}{x^2+y^2}$ c) $\frac{x}{x^2+y^2}$ d) $\frac{xy}{x^2+y^2}$

ii) The function $f(x,y)=\frac{\sqrt{x}+\sqrt{y}}{x+y}$ is a homogenous function of degree

a) 0   b) 1   c) $\frac{1}{2}$ d) $\frac{1}{n}$

iii) A sequence is said to be strictly increasing if

a) $a_n \le a_{n+1}$ b) $a_n < a_{n+1}$ c) $a_n > a_{n+1}$ d) $a_n \ge a_{n+1}$

iv) Gamma function is defined by $\Gamma(u)=$

a) $\int_0^\infty x^{u-1}e^{-x}dx$ b) $\int_0^\infty x^{u+1}e^{-x}dx$ c) $\int_0^\infty x^{u-1}e^{x}dx$ d) None

v) The improper integral is integral with

a) Infinite intervals   b) finite intervals   c) open interval   d) closed interval

vi) The correct answer to evaluate $\int \frac{e^{tanx}}{1+x^2}dx$ is

a) $e^{tan^{-1}x}$ b) e^x c) $\frac{1}{1+x^2}$ d) $\frac{e}{1+x^2}$

vii) The directional derivative of u in direction of $\vec{V}$ at the point (x_0, y_0) is given by

a) $\frac{\nabla u.\vec{V}}{V}$ b) $\frac{\nabla u.\nabla\vec{V}}{V}$ c) $\frac{\nabla\vec{V}.u}{V}$ d) $\frac{\nabla\vec{V}.u}{u}$

viii) The $\int_a^b f(x)dx$ is

a) improper integral   b) definite integral   c) indefinite integral   d) not an integral

ix) The series $\sum_{n=1}^{\infty}\frac{1}{n^n}$

a) Converges   b) Diverges   c) Both of these   d) None

x) A critical point at which f does not have a relative extrema is called

a) Stationary point   b) inflection point   c) saddle point   d) none of these

UNIVERSITY OF THE PUNJAB

Second Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus-II
Course Code: MATH-123 /

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Q.2 **Short questions** **[2x10]**

i) Evaluate the integral $\int_{-3}^{3}|x|dx$.

ii) Find equation of tangent of $xy = c^2$ at $\left(cp, \frac{c}{p}\right)$.

iii) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (y + y^3)\ dydx$

iv) Find area under between the curve $y = x^2 + 2$ and $y = 6$.

v) Find the critical points of $f(x, y) = x^2 + y^2 - axy,\ a > 0$.

vi) Integrate $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

vii) What is the difference between stationary point and critical point?

viii) Show that for Gamma function $x\Gamma(x) = \Gamma(x + 1)$.

ix) Find $\frac{dy}{dx}$, if $y = \int_0^x (t^3 + 2t + 1)dt$.

x) Evaluate $\int_{-\infty}^{0} \frac{dx}{(2x-1)^3}$.

Long questions **[5x6=30]**

Q.3 Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$

Q.4 Determine the convergence of the integral $\int_{-\infty}^{\infty} \frac{xdx}{\sqrt{x^2+2}}$.

Q.5 Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of $y^2z^3 - xy + yz + y^3 - 2x = 0$ at (1, 1, 1).

Q.6 Determine whether the following series are convergent or divergent.

(i) $\sum_{n=1}^{\infty} \left(\frac{n!}{n^n}\right)^n$ (ii) $\sum_{n=1}^{\infty} \frac{1}{1+9n^2}$

Q.7 Integrate $\int e^x \sin x\, dx$.

Q.8 State and prove the first fundamental theorem of integral calculus.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Analytical Geometry
Course Code: MATH-124 /

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Q1. Tick on the correct option (10)

i. The distance between the points $A(3,2,4)$ and $B(6,10,-1)$ is -----------.

a) 7 b) $7\sqrt{2}$ c) $2\sqrt{7}$ d) 2

ii. The three points $P(1,5,0)$, $Q(6,6,4)$ and $R(0,9,5)$ are the vertices of ---------

a) Tetrahedron b) Rectangle c) Isosceles triangle d) Parallelepiped

iii. The parametric equations of the line passing through (x_1, y_1, z_1) and parallel to the vector [**a**,**b**,**c**] are

a) $x = at,\ y = bt,\ z = ct$ b) $x - x_1 = at,\ y - y_1 = bt,\ z - z_1 = ct$

b) c) $x_1 = at,\ y_1 = bt,\ z_1 = ct$ d) $x = bt, y = ct, z = at$

iv. The distance of the point $(3,1,-2)$ to the plane $4x - 3y + 5 = 0$ is -----------

a) 13 b) $\frac{14}{5}$ c) 26 d) $\sqrt{14}$

v. The acute angle between the planes $2x + y - z - 5 = 0$ and $x - y - 2z + 5 = 0$ is ----------

a) 30° b) 45° c) 60° d) 75°

vi. If measures of the direction angles of a straight line are α, β, γ then----------

a) $sin^2\alpha + sin^2\beta + sin^2\gamma = 2$ b) $sin^2\alpha + sin^2\beta + sin^2\gamma = 1$

b) c) $sin^2\alpha + sin^2\beta + sin^2\gamma = -1$ d) $sin^2\alpha + sin^2\beta + sin^2\gamma = 0$

vii. The two planes $4x - 8y + 12z - 1 = 0$ and $2x - 4y + 6z + 2 = 0$ are --------------

a) perpendicular b) parallel c) both a and b d) None of these

viii. The equation $x^2 + y^2 - z^2 = 16$ in cylindrical coordinates is

a) $r^2 = 4$ b) $r^2 + z^2 = 16$ c) $r^2 - z^2 = 4$ d) $r^2 - z^2 = 16$

ix. The equation $x + y = 2$ represents ---------

a) Circle b) parabola c) ellipse d) surface

x. The equation $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents

a) an ellipse b) a parabola c) a hyperbola d) a circle

UNIVERSITY OF THE PUNJAB

Second Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Analytical Geometry
Course Code: MATH-124 /

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Subjective Type

Q#2: Answer the following short questions. **(5×4=20)**

a) Under what conditions on x, y and z is the point $P(x,y,z)$ equidistant from the points $(3,-1,4)$ and $(-1,5,0)$?

b) If measures of two of the direction angles of a straight line are $45°$ and $60°$, find measure of the third direction angle.

c) Find an equation of the plane through $(5,-1,4)$ and perpendicular to each of the planes $x+y-2z-3=0$ and $2x-3y+z=0$.

d) Find the rectangular coordinates of the point whose spherical coordinates are $(5, \pi/2, \pi/2)$.

e) Find the distance of the point $A(3,-1,2)$ to the plane $2x+y-z-4=0$.

Q#3: Attempt the following long questions. **3×10=30**

a) Find equations of the perpendicular from the point $P(1,6,3)$ to the straight line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also obtain its length and coordinates of the foot of the perpendicular.

b) Find the equation of the plane which passes through the point $(3,4,5)$, has an x-intercept equal to -5 and is perpendicular to the plane $2x+3y-z=8$.

c) Find an equation of the plane containing the line $x=2t,\ y=3t,\ z=4t$ and the intersection of the planes $x+y+z=0$ and $2y-z=0$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Calculus (IT)-II
Course Code: MATH-132 / ...

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Objective Type

Q. 01 **Chose the best answer.** **[10]**

i) The distance of the point $(3,1,-2)$ to the plane $4x-3y+5=0$ is -----------

a) 13 b) $\frac{14}{5}$ c) 26 d) $\sqrt{14}$

ii) The acute angle between the planes $2x+y-z-5=0$ and $x-y-2z+5=0$ is -----

a) 30° b) 45° c) 60° d) 75°

iii) If x,y, and z are independent variables and $f(x,y,z)=x\sin(y+3z)$, then f_y is

a) $-3x\cos(y+3z)$ b) $3x\cos(y+3z)$ c) $3\cos(y+3z)$ d) $x\cos(y+3z)$

iv) The gradient vector of $f(x,y)=x^2\sin 2y$ at the point $(1, \pi/2)$ is

a) $0\hat{i}+2\hat{j}$ b) $0\hat{i}-2\hat{j}$ c) $\hat{i}+2\hat{j}$ d) $-\hat{i}+2\hat{j}$

v) A spherical coordinate equation for the cone $z=\sqrt{x^2+y^2}$ is

a) $\phi=\frac{\pi}{2}$ b) $\phi=\frac{\pi}{3}$ c) $\phi=\frac{\pi}{4}$ d) $\phi=\frac{\pi}{6}$

vi) A surface presented by the equation $\rho=4$ is a

a) cone b) sphere c) line d) plane

vii) The curvature of a straight line is ....

a) zero b) one c) two d) infinite

viii) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$ represents a surface of the type

a) a paraboloid b) an elliptic cone c) a hyperboloid d) a cylinder

ix) The domain of function $w=xy\ln z$ is

a) $(x,y,z)\neq(0,0,0)$ b) Entire Space c) half space $z>0$ d) half space $z<0$

x) The equation $x^2+y^2-z^2=16$ in cylindrical coordinates is

a) $r^2=4$ b) $r^2+z^2=16$ c) $r^2-z^2=4$ d) $r^2-z^2=16$

UNIVERSITY OF THE PUNJAB

Second Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-II
Course Code: MATH-132

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

	Short Questions	
Q.02	**(a)** Find the center and radius of the sphere $x^2+y^2+z^2+4x-4z=0$. **(b)** Evaluate $\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2}} dz\,dy\,dx$ **(c)** State Stokes Theorem and define curvature function of the curve. **(d)** State divergence theorem and find $\frac{\partial z}{\partial x}$ if the equation $yz-\ln z=x+y$ defines z as a function of the two independent variables x and y and the partial derivative exists. **(e)** Find the vector projection of $\underline{u}=5\hat{j}-3\hat{k}$ onto $\underline{v}=\hat{i}+\hat{j}+\hat{k}$ and the scalar component of $\underline{u}$ in the direction of $\underline{v}$.	**[20]**
	Long Questions	
Q.03	**(a)** Find the distance from the point $(2,-3,4)$ to the plane $x+2y+2z=13$. **(b)** Show that the curvature of a circle of radius a is $\frac{1}{a}$.	**[10]**
Q.04	Derive the equation for plane in vector form and find an equation of the plane passing through $(5,-1,4)$ and perpendicular to each of the planes $x+y-2z-3=0$ and $2x-3y+z=0$.	**[10]**
Q.05	Evaluate $\int_C (x-y)dx+(x+y)dy$ counterclockwise around the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$.	**[10]**

UNIVERSITY OF THE PUNJAB

Third Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-III
Course Code: MATH-201

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Short Questions (2x10=20)

Q2.

i. Define a "Nilpotent matrix"

ii. Define a "Hermition matrix"

iii. Let $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}, |A| \neq 0, A \neq I$, then find a ,b and c if A is involutary.

iv. If A is a 3x3 matrix with detA = 2 , then find det A^5.

v. Prove that $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & a & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

vi. Show that the set {(1,3),(2,5)} is linearly independent but the set {(1,3),(2,6)} is linearly dependent in R^2.

vii. Check whether W= {(x,y,z) : x+y+z=0} is a subspace of R^3.

viii. Find the dimension of subspace R^4, spanned by {(-1,-1,5,0),(0,0,0,1)}.

ix. Find the eigen values for the linear transformation $T:R^2 \to R^2$, such that

$$T(x,y) = (3x+3y, x+5y)$$

x. Show that A and A^t have same eigen values, where A is a square matrx.

Subjective Questions (6x5=30)

Q3. Use Gauss-Jordan method to reduce the given system to reduced echelon form and determine solution.

$$6x - 6y + 6z = 6$$
$$2x - 4y - 6z = 12$$
$$10x - 5y - 5z = 30$$

Q4. If A and B are 3x3 matrices such that $\det(A^2B^2) = 108$ and $\det(A^2B^3) = 72$, find det(2A) and $\det(B^{-1})$.

Q5. Determine 'k' so that the vectors (1,-1,k-1) , (2,k,-4), (0,2+k,-8) in R^3 are linearly independent.

Q6. Let U and W be 2-dimensional subspaces of R^3, show that $U \cap W = \{0\}$.

Q7. Show that the linear transformation $T:R^2 \to R^2$ defined by

$$T(x,y) = (x+y, x-y, x+2y)$$

Is one-one.

Q8. Find an orthogonal matrix whose first row is $\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Mathematics A-III **TIME ALLOWED: 30 mins.**

Course Code: MATH-201 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE

Multiple choice questions. **(1x10=10)**

Q1. Encircle the correct answer.

i) A square matrix A is said to be periodic if

a) A^2=A b) A^{k+1}= A

c) $A^k = 0$ d) A^k =A

ii) The number of non-zero rows in echelon form of a matrix A determine

a) The rank of A b) the period of A

c) A is nilpotent d) A is idempotent

iii) Matrix A is involu-tary , if A^2=

a) I b) 0

c) A d) none of these

iv) If A is an nxn matrix and λ is a scalar, than $\det(\lambda A)$ =

a) 0 b) 1

c) $\lambda^n detA$ d) $\lambda\ detA$

v) The system of linear equations AX=B with m=n has a unique solution, if A is

a) singular b) non-singular

c) periodic d) equal to B

vi) The empty set Φ of a vector space V(F) is always taken as

a) linearly independent b) linearly dependent

c) basis d) subspace

vii) Let U be a non-empty subset of a vector space V(F), then U is said to be a basis of V(F) if

a) U is linearly independent subset of V(F) b) U spans V(F)

c) U is linearly dependent subset of V(F) d) both a and b

viii) The number m of vectors in the basis of a vector space V(F) is called the

a) Dimension of V(F) b) subspace

b) spanning set d) dimension of m

ix) A linear transformation that is both one-one and onto is called

a) Homomorphism b) isomorphism

c) Bijective d) none of these

x) Non-zero eigen vectors of a matrix A corresponding to distinct eigen values are

a) linearly independent b) linearly dependent

c) orthogonal d) none of these

UNIVERSITY OF THE PUNJAB

Third Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-III [Calculus (II)] **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-202. **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Section II (Short Questions) $(4 \times 5 = 20)$

Q 2.

(i) Find rate of change of u in the direction of v at P, where
$u = ye^{-x}(x^2 + y^2 + z^2 + 1)$, $P(0,0,0)$, $v = [2,1,2]$.

(ii) Find equation of tangent plane of $9x^2 + 4y^2 - z^2 = 36$ at $P(2,3,6)$.

(iii) Find the volume of right circular cone having base radius r and height h.

(iv) Evaluate $\int_0^{2\pi}\int_0^{1-\cos\theta} r^3\cos^2\theta\, dr\, d\theta$.

(v) Find area of surface of revolution generated by revolving about y-axis the area enclosed by the arc $x = y^3$, from $y = 0$ to $y = 1$.

Section III (Long Questions) $(6 \times 5 = 30)$

Q 3. Find area out side the circle $r = 3$ and inside the cardioid $r = 2(1 + \cos\theta)$.

Q 4. Evaluate $\int_0^4\int_0^{\sqrt{4y-y^2}}(x^2 + y^2)\,dx\,dy$ by changing into polar coordinates.

Q 5. Find the centre of gravity of a plate in the form of the segment cut from the parabola $y^2 = 8x$ by its latus rectum $x = 2$, if the density varies as the distance varies as the distance from the latus rectum.

Q 6. Find the volume of the solid bounded above by $z = 4 - x^2 - y^2$ and below by $z = 4 - 2x$.

Q 7. Determine the convergence or divergence of the series
$1 + \frac{1.2}{1.3} + \frac{1.2.3}{1.3.5} + \frac{1.2.3.4}{1.3.5.7} + \ldots$, by applying any appropriate test.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-III [Calculus (II)]
Course Code: MATH-202

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

OBJECTIVE

Section I (Multiple Choice Questions) $(1 \times 10 = 10)$

Q.1: Tick the correct answer. Cutting, Erasing or over writing is not allowed.

i) The line about which plane area revolved is called axis of ———.

a) symmetry b) revolution c) generation d) coordinates

ii) The solid generated by revolving a continuous curve about the x-axis in an interval $[a,b]$(by disc method) is ———.

a) $\pi \int f(x)dx$ b) $\pi \int [f(x)]^2 dx$ c) $\frac{4}{3}\pi \int [f(x)]^2 dx$ d) $\frac{1}{3}\pi \int [f(x)]^2 dx$

iii) The volume of a solid by washer method is calculated when $f(x) \geq g(x)$ by the formula ———.

a) $\pi \int_a^b [[g(x)]^2 - [f(x)]^2]dx$ b) $\pi \int_a^b [[f(x)]^2 + [g(x)]^2]dx$

c) $\pi \int_a^b [[f(x)]^2 [g(x)]^2]dx$ d) $\pi \int_a^b [[f(x)]^2 - [g(x)]^2]dx$

iv) Area of surface of revolution revolved about x-axis is calculated by the formula ———.

a) $\pi \int_a^b xds$ b) $2\pi \int_a^b xds$ c) $2\pi \int_a^b yds$ d) $\frac{\pi}{2} \int_a^b xds$

v) If mass of an element in xy-plane be $dm = \delta(x,y)dydx = \delta(x,y)dA$, then centre of gravity $G(\bar{x},\bar{y})$, where $\bar{x}$ is equal to ———.

a) $\frac{\int\int y\delta(x,y)dA}{\int\int \delta(x,y)dA}$ b) $\frac{\int\int x\delta(x,y)dA}{\int\int \delta(x,y)dA}$ c) $\frac{\int\int z\delta(x,y)dA}{\int\int \delta(x,y)dA}$ d) $\frac{\int\int \pi\delta(x,y)dA}{\int\int \delta(x,y)dA}$

vi) If $\sum_1^\infty a_n$ and $\sum_1^\infty b_n$ be series of positive terms with $a_n \leq b_n$ and $\sum_1^\infty b_n$ converges, then $\sum_1^\infty a_n$———.

a) converges b) bounded c) not bounded d) diverges

vii) If $u = f(x,y)$, then $\frac{\partial u}{\partial r} =$ ———.

a) $\frac{\partial u}{\partial x}\frac{\partial x}{\partial r}$ b) $\frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$ c) $\frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$ d) $\frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial y}$

viii) If $u = f(x,y)$, then $du =$ ———.

a) $\frac{\partial u}{\partial y}dy$ b) $\frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial x}dx$ c) $\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ d) $\frac{\partial u}{\partial y}dy$

ix) If $u = f(x,y)$ is homogeneous of degree n, then $nf =$ ———.

a) $xf_x + yf_y$ b) $xf_y + yf_x$ c) $f_{xy} + f_{yx}dx$ d) $f_{xx} + f_{yy}dx$

x) In the equation of plane $Ax + By + Cz + d = 0$, [A, B, C] are

a) direction ratio b) direction vector c) Normal vector d) vector at 45°

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Graph Theory
Course Code: MATH-205

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

(OBJECTIVE)

Q. # 1: Encircle the correct option. **(1x10=10)**

I. The number of vertices of 3- cubes (Q_3) graph.

(a) 8 (b) 9 (c) 12 (d) 10

II. Loop at a vertex increased the degree of the vertex by.

(a) 1 (b) 3 (c) 2 (d) 0

III. A graph in which there is a path between every two vertices in called.

(a) Disconnected (b) connected (c) Simple (d) None

IV. A complete graph with four vertices is denoted by.

(a) $K_{3,1}$ (b) $K_{2,2}$ (c) C_4 (d) K_4

V. The number of edges of a wheel graph W_3 is.

(a) 6 (b) 3 (c) 9 (d) 4

VI. For a cycle graph C_n.

(a) $n\geq0$ (b) $n\geq3$ (c) $n\geq1$ (d) $n\geq2$

VII. A tree with 6 vertices has edges.

(a) 5 (b) 6 (c) 4 (d) 7

VIII. The number of edges of a complete bipartite graph $K_{3,4}$ is:

(a) 7 (b) 12 (c) 9 (d) 16

IX. Degree of a pendant vertex is.

(a) 0 (b) 3 (c) 2 (d) 1

X. For handshaking lemma: twice the number of edges in equal to the sum of the degree, of.

(a) Edges (b) Vertices (c) Path (d) both a,b

UNIVERSITY OF THE PUNJAB

Third Semester 2015

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Graph Theory
Course Code: MATH-205

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

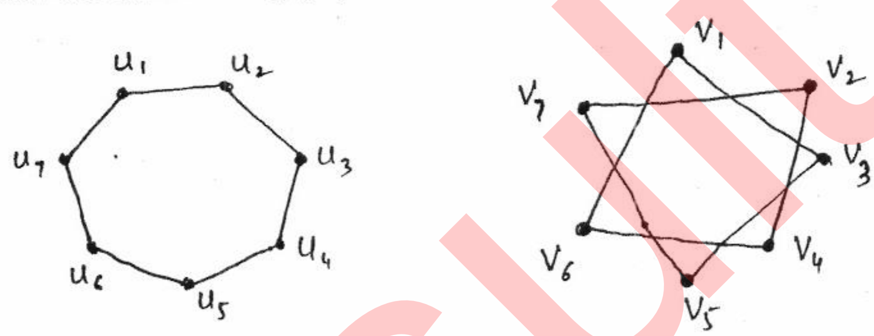
SUBJECTIVE

Q. # 2: Short Questions. **(2x10=20)**

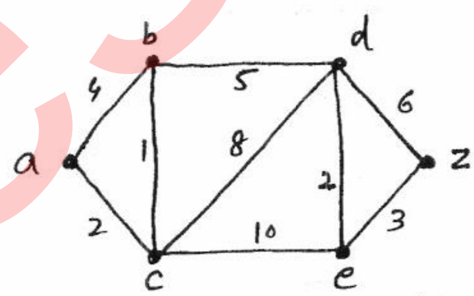
I. Draw a graph K_6.
II. How many edges are there in a graph with 4 vertices each of degree 3?
III. Find incidence matrix of a graph C_4.
IV. Define directed graph. Give two examples.
V. Write the name of four types of a graph.
VI. Draw an Euler Graph, justify by definition.
VII. What are the number of edges and vertices for n-cubes graph (Q_n).
VIII. Define isomorphism of graphs.
IX. For a full-ary tree with 5 internal vertices find the number of vertices.
X. Draw undirected graph whose adjacency matrix is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Q. # 3: Long Questions. **(6x5=30)**

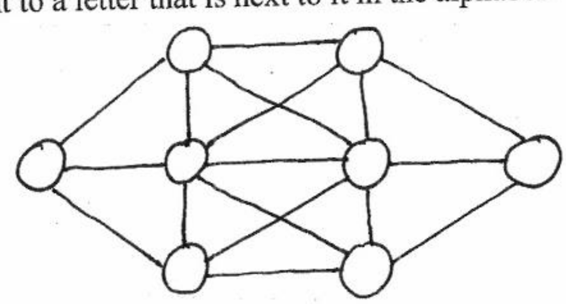
(i) For a wheel graph W_4, find the adjacency matrix and incidence matrix.
(ii) Show that the following graphs are isomorphic.



(iii) Show that there are (n-1) edges in a tree with n vertices.
(iv) Find the length of the shortest path between 'a' and 'z' in the given weighted graph.



(v) Draw a complete bipartite graph $K_{4,3}$. How many vertices and edges of this graph. Verify the Handshaking Lemma.
(vi) Place the letters A,B,C,D,E,F,G,H, into the eight circles in such a way that no letter is adjacent to a letter that is next to it in the alphabet.



UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-II (Calculus)
Course Code: MATH-211

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Note: Attempt All questions. Use of Scientific Calculators and Statistical tables is allowed but exchange of any thing i.e calculators etc. is not allowed.

	Objective Type	Marks
Q1	**Tick on the correct option**	**10**

(i) If $f(x)=-(x^2+2)$, then $f(-2)=$ ....

(a) 4 (b) 6 (c) -6 (d) -4

(ii) The solution of absolute inequality $|x-5|<1$ is

(a) $[4,6)$ (b) $(4,6)$ (c) $(-4,-6)$ (d) $(4,-6)$

(iii) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}=$ ....

(a) 0 (b) 1 (c) 2 (d) 3

(iv) $\dfrac{1}{\sqrt{1-x^2}}$ is the derivative of

(a) $\sin^{-1}x$ (b) $\tan^{-1}x$ (c) $\sec^{-1}x$ (d) $\cot^{-1}x$

v) $\int(x^2-3x)dx=$ ....

(a) $\dfrac{x^3}{3}-\dfrac{3x^2}{2}+c$ (b) $\dfrac{x^3}{3}+\dfrac{3x^2}{2}+c$ (c) $-\dfrac{x^3}{3}+\dfrac{3x^2}{2}+c$ (d) $2x-3+c$

(vi) $\int\dfrac{2x-1}{x^2-x+1}dx=$ ....

(a) $\ln(2x-1)+c$ (b) $(2x-1)+c$ (c) $x^2-x+1+c$ (d) $\ln(x^2-x+1)+c$

vii) If $f(x)=\sin x$ and $g(x)=\dfrac{1}{\tan x}$, then $fog(x)=$ ....

a) $\cot(\sin x)$ b) $\sin(\tan x)$ c) $\sin(\cot x)$ d) $\cos(\tan x)$

viii) $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx}=$

a) $\dfrac{1}{ab}$ b) 0 c) $\dfrac{a}{b}$ d) $\dfrac{a}{a}$

(ix) $\int_1^2 3x^2dx=$ ....

(a) 1 (b) 3 (c) 5 (d) 7

(x) $f(x)=\sin x$ is an increasing function for the domain $(-\pi,\pi)$ in the interval

(a) $[-\frac{\pi}{2},\frac{\pi}{2}]$ (b) $[-\frac{\pi}{2},\pi]$ (c) $[\frac{\pi}{2},\pi]$ (d) $[-\pi,\frac{\pi}{2}]$

UNIVERSITY OF THE PUNJAB

Third Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-II (Calculus)
Course Code: MATH-211

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

	Short Questions	
Q2	I. Evaluate $\int(3x^6 - 2x^2 + 7x + 1)dx$.	**10×2**
	II. Differentiate $\frac{x^2+2x+1}{x^2-2x+1}$ with respect to x.	
	III. Evaluate $\int_0^1 \frac{dx}{x^2+9}$.	
	IV. Find the second derivative $\frac{d^2y}{dx^2}$ if $y = 9x^4 + 6x^2 + 1$.	
	V. Evaluate $\lim_{x\to 3}\frac{x-3}{x^2-9}$.	
	VI. Find the average rate of change of the function $f(x) = x^2$ over the interval $[-1, 1]$.	
	VII. Evaluate $\int \ln x\, dx$	
	VIII. For what value of k $\lim_{x\to 1} f(x)$ exists, where $f(x) = \begin{cases} 2kx, & \text{if } x < 1 \\ 6-2kx, & \text{if } x > 1 \end{cases}$	
	IX. Evaluate $\int_0^{2\pi} \sin^2 x\, dx$	
	X. Evaluate $\int \frac{t\sqrt{t}+\sqrt{t}}{t^2}dt$	
	Long Questions	
Q3	If $y = \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$, then find $\frac{dy}{dx}$ by using logarithmic differentiation.	**10**
Q4	a) Evaluate $\int e^x \sin x dx$	**10**
	b) Evaluate $\lim_{y\to 0}\frac{\sin 3y \cot 5y}{y \cot 4y}$.	
Q5	a) $\int_{2/\sqrt{3}}^{2} \frac{\cos(\sec^{-1} x)dx}{x\sqrt{x^2-1}}$	**10**
	b) $\int_0^{\pi/4} \sqrt{1+\cos 4x}\, dx$	

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Differential Equations-I
Course Code: MATH-221

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

SECTION – I (Objective)

Marks=10

(i). The particular solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = 2x^2 - 3x + 6$ is

(a). $y_p = -x^2 - \frac{5}{2}x - 9$
(b). $y_p = x^2 + \frac{5}{2}x + 9$
(c). $y_p = -x^2 + \frac{5}{2}x - 9$
(d). $y_p = 0$

(ii). The differential equation $\frac{d^2x(t)}{dt^2} + \omega^2 \sin x(t) = F_0 \sin(\omega t)$, where ω is a constant (natural frequency of the oscillator)

(a). is not linear, there is an ω^2 factor in front of $\sin x(t)$
(b). is not linear, due to the factor $\sin x(t)$
(c). the differential equation is a linear
(d). None of above

(iii). The method of undetermined coefficients allows us to write particular solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 8e^{3x} + 4\sin x$ is

(a). $y_p = Ae^{3x} + B\cos x + C\sin x$
(b). $y_p = Ae^{3x} + B\cos 2x + C\sin 2x$
(c). $y_p = Axe^{3x} + B\cos 2x + C\sin 2x$
(d). $y_p = Axe^{3x} + B\cos x + C\sin x$

(iv). The differential equation $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^2 - 4y = e^x$

(a). is not linear, due to the factor $\left(\frac{dy}{dx}\right)^2$
(b). is not linear, due to the factor e^x
(c). is not linear, due to the factors $\frac{d^2y}{dx^2}$
(d). None of above

(v). The differential equation $\frac{d^2y}{dx^2} + \omega^2 \sin y = 0$,

(a). is a second order linear
(b). is a second order nonlinear
(c). is a second order exact
(d). both (a) and (c)

(P.T.O.)

(vi). The differential operator $L = \left(\frac{d^2}{dx^2} + a^2\right)^2$ annihilates (i.e. $Ly = 0$) the function

(a). $y(x) = a$
(b). $y(x) = \cos(ax)$
(c). $y(x) = \sin(ax)$
(d). both $y(x) = x\sin(ax)$ and $y(x) = x\cos(ax)$

(vii). The differential equation $\frac{dy}{dx} + P(x)y = f(x)y^n; \quad n \neq 0, 1,$

(a). is called Bernoulli's equation
(b). is called Riccati equation
(c). is called first order linear differential equation
(d). is called an exact differential equation

(viii). A first-order differential equation of the form

$$M(x,y)dx + N(x,y)dy = 0,$$

(a). is a linear equation
(b). is exact differential equation if $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$
(c). is exact differential equation if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
(d). is exact differential equation if $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(ix). The one-parameter family $y(x) = cx^3$, is a one-parameter family of solutions of the differential equation

(a). $x\frac{dy}{dx} - 3y = 0$
(b). $x\frac{dy}{dx} + 3y = 0$
(c). $x\frac{dy}{dx} - 3 = 0$
(d). $\frac{dy}{dx} - 3y = 0$

(x). If L is a linear differential operator such that $L(y_1) = 0$ and $L(y_2) = 0$

(a). $L(c_1y_1 + c_2y_2) = 0$
(b). $L(c_1y_1 + c_2y_2) \neq 0$
(c). $L(c_1y_1y_2 + c_2y_1y_2) = 0$
(d). both (a) and (c)

UNIVERSITY OF THE PUNJAB

Third Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Differential Equations-I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-221 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Section-II (Short Questions) Marks=20

1. Verify that the piecewise-defined function

$$y = \begin{cases} -x^2; & x < 0 \\ x^2; & 0 \le x \end{cases}$$

is a solution of the differential equation $x\frac{dy}{dx} - 2y = 0$, on the interval $(-\infty, \infty)$.

2. Solve the initial-value problem

$$\frac{dy}{dx} - 2xy = 2, \qquad y(0) = 1.$$

3. Show that the $\{5, \sin^2 x, \cos^2 x\}$ is linearly dependent on the interval $(-\infty, \infty)$.

4. Given that $y_1(x) = e^x$ is a solution of $\frac{d^2y}{dx^2} - y = 0$ on the interval $(-\infty, \infty)$, use reduction of order to find a second solution $y_2(x)$.

5. Solve the initial-value problem

$$\frac{dy}{dx} = (-2x + y)^2 - 7, \qquad y(0) = 0,$$

by using an appropriate substitution.

Section-III Marks=30

1. Solve the differential equation by using undetermined coefficients

$$\frac{d^2y(x)}{dx^2} - \frac{dy(x)}{dx} + y(x) = 2\sin(3x).$$

2. Solve the given differential equation

$$x\frac{dy}{dx} + y = x^2y^2.$$

3. Solve

$$(x^2 + y^2)dx + (x^2 - xy)dy = 0.$$

4. Solve the system of linear first-order differential equations

$$\begin{aligned} \frac{dx_1}{dt} &= -\frac{2}{25}x_1 + \frac{1}{50}x_2, \\ \frac{dx_2}{dt} &= \frac{2}{25}x_1 - \frac{2}{25}x_2. \end{aligned}$$

5. First verify by substitution that $y_1(x) = x^{-1/2}\cos x$ and $y_2(x) = x^{-1/2}\sin x$ are two linearly independent solutions of associated homogeneous Bessel's differential equation, i.e.,

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \frac{1}{4})y = 0.$$

Find the general solution of the nonhomogeneous Bessel's equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \frac{1}{4})y = x^{3/2}.$$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2015
Examination: B.S. 4 Years Programme

PAPER: Pure Mathematics
Course Code: MATH-222/

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q1. Encircle the correct option. **(1*10=10)**

i). The set $A=\{x\in\mathbb{R}:x\neq x\}=$

a). $\{0\}$ b). $\{\mathbb{R}\}$ c). $\mathbb{R}$ d). ϕ

ii). The set ϕ is an element of the set

a). $\{x\}$ b). ϕ c). $\{\phi\}$ d). $\{\{\phi\}\}$

iii). If $f(x)=2x-1$ then $f^{-1}(x)=$

a). $(x+1)/2$ b). $(x-1)/2$ c). $(2x+1)/2$ d). $(2x-1)/2$

iv). The converse of the conditional $\sim p\rightarrow q$ is

a). $p\rightarrow\sim q$ b). $q\rightarrow\sim p$ c). $q\rightarrow p$ d). $\sim p\rightarrow\sim q$

v). If U is the universal set. Then for any set $A\neq\phi$, $(A^c)^c=$

a). U b). ϕ c). A d). A^c

vi). The indiscrete topology on $\mathbb{R}$ is given by

a). $\tau=\{\phi,\mathbb{R}\}$ b). $\tau=P(\mathbb{R})$ c). $\tau=\{\phi,Q,Q^*,\mathbb{R}\}$ d). $\tau=\{\phi,\mathbb{R},\{1\}\}$

vii). Let $X=\{a,b,c,d,e\}$ with $\tau=\{X,\phi,\{b\},\{d\},\{b,d\}\}$. Let $A=\{a,c,e\}$. Then $\overline{A}=$........

a). A b). A^c c). ϕ d). X

viii). The set $X=\{a,b\}$, with the topology $\tau=\{\phi,X,\{a\}\}$ is called -----space

a). *cofinite* b). *euclidean* c). *sierpinski* d). *None*

ix). A set X with one element has.......topology (topologies).

a). *four* b). *three* c). *two* d). *one*

x). If $X=\{1,2,3,4\}$, then a partition of X is

a). $\{1,2\},\{3\},\{4\}$ b). $\{1\},\{2\},\{3\}$ c). $\{1,2\},\{1,4\},\{1,3\}$ d). $\{1,2,3\},\{3,4\}$

UNIVERSITY OF THE PUNJAB

Third Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Pure Mathematics
Course Code: MATH-222/

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q2. Answer the following short questions. **(2*10=20)**

i). If $f(x)=(x-2)/4$ and $g(x)=4x$. Calculate the composition $g \circ f$ and hence $g \circ f(0)$.

ii). Define bijective function. Give two examples.

iii). Define discrete metric space.

iv). Define finer and coarser topologies. Give one example.

v). Define conjunction and disjunction.

vi). Define absurdity. Give two examples.

vii). Let $\tau = \{\phi, \mathbb{R}, \mathbb{Q}, \mathbb{Q}^*\}$ be a topology on $\mathbb{R}$. Find all the neighborhoods of 2.

viii). Let $X = \{a,b,c,d\}$. Find the topology generated by the base $\beta = \{\{a,b\},\{b,c\},\{b\},\{d\}\}$.

ix). In a topological space (X,τ) for any subsets A, B of X, $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$.

x). Let $\tau = \{\phi, \mathbb{R}, \{1\}, \{2\}, \{1,2\}, [1,\ 2]\}$ be a topology on $\mathbb{R}$. Find all the closed subsets of $\mathbb{R}$.

Long Questions **(6*5=30)**

Q3. Let (X,τ) be a topological space. Then show that

i) a subset A *is closed* $\Leftrightarrow \overline{A} = A$ ii). a subset A *is open* $\Leftrightarrow A^o = A$

Q4. Let $X = \{1,2,3,4,5,6,7,8\}$. Find five topologies on X, each containing four members.

Q5. Show that the statement $\neg(p \to q) \leftrightarrow (p \wedge \neg q)$ is tautology using truth table.

Q6. Use mathematical induction to show that for every natural number n,

$$1+3+5+\dots\dots+(2n-1)=n^2$$

Q7. Prove that any open ball in the usual metric space $\mathbb{R}$ is an open interval. Give one example of an open set which is not an open interval.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Discrete Mathematics (IT)
Course Code: MATH-231

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

OBJECTIVE

Q1 Tick on the correct option — 10

i) $\neg p \vee \neg q$ is logically equivalent to

a) $\neg p \wedge \neg q$ b) $\neg(p \wedge q)$ c) $\neg p \wedge q$ d) $p \wedge \neg q$

ii) Number of edges in K_4 are

a) 4 b) 5 c) 6 d) 8

iii) Number of strings can be made by reordering the letters of SUCCESS.

a) 340 b) 420 c) 512 d) 625

iv) How many permutations of letters ABCDE contain the string ABC.

a) 3! b) 4! c) 5! d) 6!

v) Graphs that a number assigned to each edge are called...............graphs

a) complete b) weighted c) simple d) bipartite

vi) The cardinality of the set $A = \{a, \{a\}, \{a, \{a\}\}\}$ is

a) 3 b) 4 c) 2 d) 1

vii) The set $P(\{a, b, \{a, b\}\})$ has elements

a) 4 b) 8 c) 12 d) 16

viii) $\overline{A \cup (B \cap C)} =$

a) $(\overline{C} \cup \overline{B}) \cap \overline{A}$ b) $(\overline{A} \cup \overline{B}) \cap \overline{C}$ c) $(\overline{C} \cup \overline{A}) \cap \overline{B}$ d) $(\overline{C} \cap \overline{B}) \cup \overline{A}$

ix) The domain of the function $f(x) = \sqrt{|x|}$ is

A) $(-\infty, 0]$ b) $[0, \infty)$ c) $(-\infty, \infty)$ d) all of these

x) If both f and g are one-to-one functions, then $f \circ g$ is

a) one-to-one b) onto c) a & b d) none of these

UNIVERSITY OF THE PUNJAB

Third Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics (IT) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-231 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Q2 **10×2**

i) Let $f: R^{+} \to R$ be defined by $f(x)=\frac{3\sqrt{x}-1}{2}$. Find a formula for f^{-1}.

ii) Prove that an undirected graph has an even number of vertices of odd degree.

iii) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

iv) Let a, b and c be positive integers. Prove that if $a|b$ and $b|c$ then $a|c$.

v) Define one-to-one and onto functions.

vi) Let $f: R \to R$ be defined by $f(x)=x^2$. Determine whether $f(x)$ is one-to-one? Is this function onto?

vii) Find the prime factorization of 45617.

viii) List five integers that are congruent to 3 modulo 11.

ix) Draw the graph of $K_{5,4}$ and W_7.

x) Let $A=\{0,1,2,3\}$ and $R=\{(0,0),(1,1),(1,3),(2,2),(2,3),(3,1),(3,2),(3,3)\}$. Show that R is an equivalence relation.

Long Questions

Q3 **10**

a) Use mathematical induction to show that $1^3+2^3+3^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2$ for all positive integers n.

b) If T is a tree with n vertices then prove that T contains no cycles, and has $n-1$ edges.

Q4 **10**

a) Show that the implication $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology by using truth table.

b) Show that the propositions $\sim \forall x(p(x) \to q(x))$ and $\exists x(p(x) \wedge \sim q(x))$ are logically equivalent.

Q5 **10**

a) Give a formula for the coefficients of x^k, k is an integer, in the expansion of $(x^2-\frac{1}{x})^{100}$.

b) Draw the graph whose adjacency matrix is given by

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fourth Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Mathematics A-IV
Course Code: MATH-203 /

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Objective Type

Q. No.1 Encircle the correct option. **(1*10)**

(i) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ is the differential equation having

a). order 1, degree 1 **b).** order 2, degree 2
c). order 1, degree 2 **d).** order 2, degree 1

(ii) The differential equation $\frac{d^2y}{dx^2}+y\frac{dy}{dx}+yx=0$ is

a). linear and ordinary **b).** linear and partial
c). nonlinear and ordinary **d).** nonlinear and partial

(iii) The solution of differential equation $\frac{dy}{dx}-e^x=0$ is

a). $y=xe^x$ **b).** $y=c+e^x$ **c).** $y=\ln x$ **d).** $y=ce^x$

(iv) Which differential equation is not exact

a). $-ydx+xdy=0$ **b).** $ydx+xdy=0$
c). $(3x^2y)dx+(x^3)dy=0$ **d).** $(2xy-3)dx+(x^2+4y)dy=0$

(v) $f(x,y)=\sqrt{xy}$ is a homogenous function of degree

a). 0 **b).** 1 **c).** 2 **d).** 1/2

(vi) The singular points of $(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ are

a). 0 and 1 **b).** 1 and -1 **c).** 0 and -1 **d).** none

(vii) The roots of characteristic equation of $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=0$ are

a). 1 and 3 **b).** -1 and 3 **c).** 1 and -3 **d).** -1 and -3

(viii) If 3 and -4 are the roots of characteristic equation, then solution of the differential equation is

a). $y=c_1e^{3x}+c_2e^{-4x}$ **b).** $y=3c_1e^x-4c_2e^x$
c). $y=(c_1+c_2e^{3x})e^{-4x}$ **d).** $y=c_1e^{3x}-c_2e^{4x}$

(ix) If y_1, y_2 are differentiable functions of x on $[0,1]$. Then their Wronskian $W[y_1,y_2]=$

a). $y_1y_2-y_1'y_2'$ **b).** $y_1y_1'-y_2y_2'$
c). $y_1y_2'-y_1'y_2$ **d).** $y_1y_2'+y_1'y_2$

(x) $y=A\sin x+B\cos x$ is the general solution of

a). $\frac{d^2y}{dx^2}-y=0$ **b).** $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$
c). $\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$ **d).** $\frac{d^2y}{dx^2}+y=0$

UNIVERSITY OF THE PUNJAB

Fourth Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-IV
Course Code: MATH-203

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Subjective Type

Q No. 2. Give short answers of the following Questions. **(2*10)**

(i) Solve the initial value problem $\frac{dy}{dx} = -x$ with $y(3) = 4$.

(ii) Define Cauchy-Euler Equation. Give one example.

(iii) Form the differential equation of the curve $y = \cos x - e^{-x}$.

(iv) Reduce the differential equation $\frac{dy}{dx} = \frac{2y - x + 5}{2x - y - 4}$ into homogeneous form.

(v) Calculate the integrating factor of the differential equation $(x^2 - 2x + 2y)dx + 2xydy = 0$.

(vi) What is the Principle of Superposition?

(vii) Calculate the C.F of $\frac{d^2y}{dx^2} - 4y = e^x + \sin x$.

(viii) Find ordinary points of $(x^2 - 5x + 6)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$.

(ix) If a population increases at a rate proportional to its current value. Form a differential equation.

(x) Define regular singular point.

Long Questions **(3*10)**

QNo3. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$ with $y(0) = 1,\ y'(0) = -8,\ y''(0) = -4$.

QNo4. Solve $2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = x^2 + 3\sin x$ by the method of Undetermined Coefficients.

QNo5. Solve $\frac{dy}{dx} + \frac{xy}{1 - x^2} = xy^{\frac{1}{2}}$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fourth Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-IV
Course Code: MATH-204

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Q. 1 **SECTION-I** **MCQs (1 Mark each)**

(i) A function $f:(R,d)\to(R,d')$ is continuous at $c\in R$ if and only if for every $\varepsilon>0$, there exists $\delta>0$ such that $d(x,a)<\delta$ implies that ----------

(a) $d'(f(x),f(c))=\varepsilon$ (b) $d'(f(x),f(c))<\varepsilon$

(c) $d'(f(x),f(c))>\varepsilon$ (d) $d'(f(x),f(c))\le\varepsilon$

(ii) In (R, d) with usual metric d on R, the set N of natural is -------- in the real line R.

(a) Open (b) Both open and closed (c) Closed (d) Neither open nor closed

(iii) The set Q^c of all irrational numbers is ---------- subset of R.

(a) Neither open nor closed (b) both open and closed

(c) Closed (d) Open

(iv) The closure of the subset $]1,2]$ of the real line R under the usual metric is ------

(a) $]1,2[$ (b) $[1,2[$ (c) $]1,2]$ (d) $[1,2]$

(v) The interior of the subset $\{1,2,3,4,5\}$ of the real line R under the usual metric is ------

(a) ϕ (b) $\{1,2,3,4,5\}$ (c) $(1,5)$ (d) $[1,5]$

(vi) Number of non-isomorphic groups of order 8 is ....

(a) 4 (b) 2 (c) 3 (d) 5

(vii) The order of the permutation $\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$ is

(a) 2 (b) 3 (c) 4 (d) 5

(viii) If $x^2=x$ for some x in a group G then x is called ----------

(a) Involution (b)element of infinite order (c) Idempotent (d) None of these

(ix) Every group of even order has at least one element of order ----------

(a) 1 (b) 2 (c) 3 (d) 4

(x) Let G be a group of order 35 then G has a subgroup of order ------

(a) 2 (b) 3 (c) 4 (d) 5

UNIVERSITY OF THE PUNJAB

Fourth Semester 2015

Examination: B.S. 4 Years Programme

Roll No. ....

TIME ALLOWED: 2 hrs. & 30 mins.

MAX. MARKS: 50

PAPER: Mathematics B-IV

Course Code: MATH-204 /

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2

(i) State and prove the Minkowski's Inequality . (2)

(ii) Find the limit points of the set $(2,7]$ in (R,d) where *d* is a usual metric on *R*. (2)

(iii) Let $u=(x_1,x_2), v=(y_1,y_2)\in R^2$ then prove that $\langle u,v\rangle = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$ defines an inner product on R^2. (2)

(iv) Define neighborhood of a point and prove that the intersection of any two neighborhoods is also its neighborhood. (2)

(v) Let (X,d) be a metric space then prove that $|d(x,z)-d(y,z)|\le d(x,y)$ for all $x,y,z\in X$. (2)

(vi) Prove that the set C of complex numbers is a group under addition. (2)

(vii) Prove that $G=\{\bar{1},\bar{3},\bar{5},\bar{7}\}$ be a group of residue classes under multiplication modulo 8. (2)

(viii) Let Z be the set of all integers then prove that $H=\{0,\pm 2,\pm 4,\pm 6,\dots\}$ is a subgroup of Z also find all left cosets of H in Z. (2)

(ix) Define index of a sub group. (2)

(x) Distinguish between cycle and transposition. (2)

SECTION-III

Q.3 Show that every open ball in a metric space (X, d) is open. (6)

Q.4 Let (X,d) be a metric space then show that $d'(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$ is also a metric on X. (6)

Q.5 State and Prove the Lagrange's Theorem. (6)

Q.6 Give an example of an abelian group which is not cyclic. (6)

Q.7 Let $X=\{1,2,3\}$ write all permutation on X. (6)

UNIVERSITY OF THE PUNJAB

Fourth Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-IV
Course Code: MATH-204 /

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Q. 2 **SECTION-II**

(i) State and prove the Minkowski's Inequality . (2)

(ii) Find the limit points of the set $(2,7]$ in (R,d) where d is a usual metric on R. (2)

(iii) Let $u=(x_1,x_2), v=(y_1,y_2)\in R^2$ then prove that $\langle u,v\rangle = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$ defines an inner product on R^2. (2)

(iv) Define neighborhood of a point and prove that the intersection of any two neighborhoods is also its neighborhood. (2)

(v) Let (X,d) be a metric space then prove that $|d(x,z)-d(y,z)| \le d(x,y)$ for all $x,y,z\in X$. (2)

(vi) Prove that the set C of complex numbers is a group under addition. (2)

(vii) Prove that $G=\{\bar{1},\bar{3},\bar{5},\bar{7}\}$ be a group of residue classes under multiplication modulo 8. (2)

(viii) Let Z be the set of all integers then prove that $H=\{0,\pm2,\pm4,\pm6,...\}$ is a subgroup of Z also find all left cosets of H in Z. (2)

(ix) Define index of a sub group. (2)

(x) Distinguish between cycle and transposition. (2)

SECTION-III

Q.3 Show that every open ball in a metric space (X, d) is open. (6)

Q.4 Let (X,d) be a metric space then show that $d'(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$ is also a metric on X. (6)

Q.5 State and Prove the Lagrange's Theorem. (6)

Q.6 Give an example of an abelian group which is not cyclic. (6)

Q.7 Let $X=\{1,2,3\}$ write all permutation on X. (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fourth Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Elementary Number Theory
Course Code: MATH-206 / ......

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1	**MCQs (Marks =10)**
(i)	The number 670040107 is divisible by a) 7 b) 11 c) 9 d) 99
(ii)	What is the least common multiple of 540 and 216? a) $2.\,3^2\,.5$ b) $2^3.\,3^3\,.5$ c) $2^2.\,3^3\,.5$ d) $2^3.\,3^4\,.5$
(iii)	M is a multiple of 96. Which of the following cannot be true? a) M is a multiple of 4 b) M is 96 c) M is 0 d) M is odd
(iv)	What is the greatest common divisor of 273 and 81? a) $2^3.3^3$ b) $2^2.3^2$ c) 3 d) $2^2.3^3$
(v)	If gcd(a,b)=15 and lcm(a,b)=12, then 1) a=18, b=10 2) a=18, b=12 3) a=12, b=10 4) a=15, b=18
(vi)	Every linear congruence contains__________ solutions a) one b) zero c) infinite d) None of (a),(b),(c)
(vii)	If $48x \equiv 90 \pmod{152}$ then number of incongruent solutions for x is a) 3 b)4 c)6 d)8
(viii)	Product of any *p* consecutive integers is divisible by a) *P!* b) *(p-1)!* c) *(p+1)!* d) None of (a),(b),(c)
(ix)	The sum of positive divisors of 28 is a) 6 b) 4 c) 7 d) 56
(x)	Femat's numbers are always a) Even b) Prime c) Perfect d) Odd

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fourth Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Elementary Number Theory
Course Code: MATH-206 /

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

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UNIVERSITY OF THE PUNJAB

Fourth Semester 2015

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Number Theory
Course Code: MATH-206 /

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2	Short Questions (2x10 = 20 Marks)
(i)	What is the remainder when 3^{100} is divided by 28?
(ii)	Find the missing digit x of the number **17x521221** if it is divisible by 3?
(iii)	Solve $111x+15y=21$
(iv)	Define Mersenn and Fermat's primes with one example of each.
(v)	Show that the Diophantine equation $x^2-4y^2=2$ has no solution.
(vi)	Find (i) lcm [273,81] (ii) gcd (275,105)
(vii)	Define Linear Congruence and write down its general solution.
(viii)	State Eculid's theorem.
(ix)	Prove that gcd(a,b). lcm(a,b)=ab for integers a and b.
(x)	If $a\equiv b(\mathrm{mod}\, m)$ and $b\equiv c(\mathrm{mod}\, m)$ then prove that $a\equiv c(\mathrm{mod}\, m)$

Section-III

	Long Questions (6x5 = 30 Marks)
Q.3	Let $n>1$ be a composite integer then show that there exists a prime p such that $p\mid n$ and $p\le\sqrt{n}$
Q.4	Solve the system of linear congruences $x\equiv 5\ (\mathrm{mod}\ 11)$ $x\equiv 2(\mathrm{mod}\ 19)$
Q.5	Express gcd(256, 1166) as a linear combination of 256 and 1166.
Q.6	Prove that any two Fermat's numbers are relatively prime to each other.
Q.7	(i) Prove that product of any four consecutive integers is divisible by 24. (ii) Prove that 24 divides $2.7^n+3.5^n-5\ \forall n>0$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fourth Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Differential Equations-II
Course Code: MATH-223 /

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Section-I (Objective) Marks=10

1. (i). $\mathcal{L}\{f(t)\} = \lim_{T\to\infty}\int_0^T f(t)e^{-st}dt$, exist for $s > c$
   - (a). if $f(t)$ is piecewise continuous on the interval $[0, \infty)$ and of exponential order c for $t > T$
   - (b). if $f(t)$ is piecewise continuous on the interval $(-\infty, \infty)$ and of exponential order c for $t > T$
   - (c). if $f(t)$ is piecewise continuous on the interval $[0, \infty)$
   - (d). if $f(t)$ is piecewise continuous on the interval $[0, \infty]$

(ii). The equation $x\frac{dy}{dx} + 6y = 3xy^{4/3}$,
   - (a). is linear first order differential equation
   - (b). is the Ricatti's differential equation
   - (c). is the Bernoulli equation
   - (d). None of above

(iii). A general solution of the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \nu^2)y = 0$,is given by
   - (a). $y = AJ_\nu(x) + BJ_{-\nu}(x)$
   - (b). $y = AJ_\nu(x) + B(xJ_{-\nu}(x))$
   - (c). $y = AJ_\nu(x) + B(xY_\nu(x))$
   - (d). $y = AJ_\nu(x) + BY_\nu(x)$

(iv). $y(x) = x^n J_n(x)$ is a particular solution of
   - (a). $x\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + xy = 0, \quad x > 0$
   - (b). $x^2\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + xy = 0, \quad x > 0$
   - (c). $x\frac{d^2y}{dx^2} + (1+2n)\frac{dy}{dx} + xy = 0, \quad x > 0$
   - (d). $x\frac{d^2y}{dx^2} - (1-2n)\frac{dy}{dx} - xy = 0, \quad x > 0$

(v). $x^2(x+1)\frac{d^2y}{dx^2} + x(4-x^2)\frac{dy}{dx} + (2+3x)y = 0$,
   - (a). $x = 0$ is irregular singular point
   - (b). $x = 0$ is an ordinary point
   - (c). $x = 0$ is a regular singular point
   - (d). None of above

(vi). The function $f(t) = \begin{cases} \sin t, & 0 \leqslant t < \pi \\ 0, & t \geqslant \pi \end{cases}$ can be expressed in term of unit step function as
   - (a). $f(t) = \sin t - \sin t\, U(t-\pi)$
   - (b). $f(t) = \sin t + \sin t U(t-\pi)$
   - (c). $f(t) = \sin t\, U(t-\pi)$
   - (d). $f(t) = \sin t + U(t-\pi)$

(vii). The Laplace transform of unit step function $U(t-\pi) =$
   - (a). $\frac{e^{\pi s}}{s}$
   - (b). $e^{-\pi s}$
   - (c). $\frac{e^{-\pi s}}{s^2}$
   - (d). $\frac{e^{-\pi s}}{s}$

(viii). $\mathcal{L}\{t^n\} =$
   - (a). $\frac{\Gamma(n+1)}{s^{n+1}}, \quad n > -1$
   - (b). $\frac{\Gamma(n-1)}{s^{n+1}}, \quad n > 1$
   - (c). $\frac{\Gamma(n+2)}{s^{n+1}}, n > -2$
   - (d). $\frac{\Gamma(n)}{s^{n-1}}, \quad n > 0$

(ix). $\mathcal{L}\{\cos kt\} =$
   - (a). $\frac{ks}{s^2+k^2}$
   - (b). $\frac{s}{s^2-k^2}$
   - (c). $\frac{s}{s^2+k^2}$
   - (d). $\frac{k}{s^2+k^2}$

(x). $\mathcal{L}[f(t) + g(t)] =$
   - (a). $\frac{F(s)}{G(s)}$
   - (b). $F(s) - G(s)$
   - (c). $F(s) + G(s)$
   - (d). $F(s)G(s)$

UNIVERSITY OF THE PUNJAB

Fourth Semester 2014
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Differential Equations-II
Course Code: MATH-223 /

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Section-II

Marks=20

1. If $F(s) = \mathcal{L}\{f(t)\}$ and $a > 0$, then show that

$$\mathcal{L}\{f(t-a)U(t-a)\} = e^{-as}F(s), \qquad \text{where } U(t-a) = \begin{cases} 0, & 0 \leqslant t < a \\ 1, & t \geqslant a \end{cases}.$$

2. Use the Laplace transform to solve the given initial-value problem (IVP)

$$\frac{dy}{dt} + 4y = e^{-4t}, \qquad y(0) = 2.$$

3. Verify that $y = x^{-n}J_n(x)$ is a particular solution of

$$x\frac{d^2y}{dx^2} + (1+2n)\frac{dy}{dx} + xy = 0, \qquad x > 0.$$

4. Find $\mathcal{L}\{f(t)\}$, where

$$f(t) = \begin{cases} \sin t, & 0 \leqslant t < \pi \\ 0, & t \geqslant \pi. \end{cases}$$

5. An equation of the form

$$y(x) = xy' + a\sqrt{1+(y')^2}, \qquad \text{where } y' = \frac{dy}{dx} \tag{1}$$

is called a Clairaut equation. Show that the one-parameter family of straight lines described by

$$y(x) = Cx + a\sqrt{1+C^2},$$

is a general solution of equation (1).

Section-III

Marks=30

1. Find two power series solutions of the differential equation

$$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0,$$

about the ordinary point $x = 0$.

2. Use the Laplace transform to solve the given initial-value problem (IVP)

$$\frac{d^2y}{dt^2} + 4y = f(t), \qquad y(0) = 0, \quad y'(0) = -1 \text{ where } f(t) = \begin{cases} 1, & 0 \leqslant t < 1 \\ 0, & t \geqslant 1 \end{cases}.$$

3. Use the Laplace transform to solve the given integrodifferential equation

$$\frac{dy(t)}{dt} + 6y(t) + 9\int_0^t y(\tau)d\tau = 1, \qquad \text{with} \qquad y(0) = 0.$$

4. Solve the Riccati differential equation

$$\frac{dy}{dx} + (\cot x)\,y - y^2 + \csc^2 x = 0,$$

given that $y_1(x) = \cot x$ is a particular solution.

5. Use the Laplace transform to solve the system of linear differential equations

$$\frac{d^2x_1(t)}{dt^2} + \frac{d^2x_2(t)}{dt^2} = t^2,$$
$$\frac{d^2x_1(t)}{dt^2} - \frac{d^2x_2(t)}{dt^2} = 4t,$$

subject to $x_1(0) = 8$, $\left.\frac{dx_1(t)}{dt}\right|_{t=0} = 0$, $x_2(0) = 0$, $\left.\frac{dx_2(t)}{dt}\right|_{t=0} = 0$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fourth Semester 2015

Examination: B.S. 4 Years Programme

PAPER: Linear Algebra
Course Code: MATH-224 / [illegible]

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Section-I (Objective) Marks=10

1. (i). If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrices of same size, then sum of matrices A and B is defined as
   - **(a).** $A + B = C$ with $c_{ij} = a_{ik} + b_{kj}$.
   - **(b).** $A + B = C$ with $c_{ij} = a_{ij} + b_{ij}$.
   - **(c).** $A + B = C$ with $c_{ij} = a_{jj} + b_{ii}$.
   - **(d).** $A + B = C$ with $c_{ij} = a_{ii} + b_{jj}$.

(ii). If $A = [a_{ij}]_{n\times n}$, then $\text{Tr}(A) =$
   - **(a).** $\sum\limits_{i}^{n}\sum\limits_{j}^{n} a_{ij}$
   - **(b).** $\sum\limits_{i}^{n} a_{ii}$
   - **(c).** $a_{11} + a_{22} + \ldots + a_{nn}$
   - **(d).** both (b) and (c)

(iii). If $\mathbf{u}$ and $\mathbf{v}$ are vectors in a real inner product space V, then
   - **(a).** $|\langle \mathbf{u}, \mathbf{v}\rangle| \leqslant \|\mathbf{u}\|\,\|\mathbf{v}\|$
   - **(b).** $|\langle \mathbf{u}, \mathbf{v}\rangle| = \|\mathbf{u}\|\,\|\mathbf{v}\|$
   - **(c).** $|\langle \mathbf{u}, \mathbf{v}\rangle| \geqslant \|\mathbf{u}\|\,\|\mathbf{v}\|$
   - **(d).** None of above

(iv). If $\mathbf{x}$ and $\mathbf{y}$ are vectors in a complex vector space W, then
   - **(a).** $\langle \mathbf{x}, \mathbf{y}\rangle = \overline{\langle \mathbf{y}, \mathbf{x}\rangle}$
   - **(b).** $\langle \mathbf{x}, \mathbf{y}\rangle = \overline{\langle \mathbf{x}, \mathbf{y}\rangle}$
   - **(c).** $\langle \mathbf{x}, \mathbf{y}\rangle = -\overline{\langle \mathbf{y}, \mathbf{x}\rangle}$
   - **(d).** $\langle \mathbf{x}, \mathbf{y}\rangle = -\overline{\langle \mathbf{x}, \mathbf{y}\rangle}$

(v). If $A = [a_{ij}]_{n\times n}$ is a diagonal matrix, then
   - **(a).** $\det(A) = (1 - a_{11})(1 - a_{22})\ldots(1 - a_{nn})$
   - **(b).** $\det(A) \neq a_{11}a_{22}\ldots a_{nn}$
   - **(c).** $\det(A) = a_{11}a_{22}\ldots a_{nn}$
   - **(d).** None of above

(vi). If A is an invertible matrix, then $\left(A^{-1}\right)^{\dagger} =$
   - **(a).** A
   - **(b).** $A^{\dagger}$
   - **(c).** $\left(A^{\dagger}\right)^{-1}$
   - **(d).** A^{-1}

(vii). If G is a group, then
   - **(a).** $(ab)^{-1} = b^{-1}a^{-1} \forall a, b \in G$
   - **(b).** $(ab)^{-1} \neq b^{-1}a^{-1} \forall a, b \in G$
   - **(c).** $(ab)^{-1} = a^{-1}b^{-1} \forall a, b \in G$
   - **(d).** both (a) and (c)

(viii). The product of two even permutations is
   - **(a).** even.
   - **(b).** odd.
   - **(c).** both (a) and (b).
   - **(d).** None of above

(ix). In a group $(Z, +)$, the additive inverse of -3 is
   - **(a).** $\frac{1}{3}$
   - **(b).** 3
   - **(c).** 0
   - **(d).** $-\frac{1}{3}$

(x). Two groups, $(G, *)$ and $(H, \cdot)$, a group homomorphism from $(G, *)$ to $(H, \cdot)$ is a function $h : G \to H$ such that for all u and v in G it holds that
   - **(a).** $h(u * v) \neq h(u).h(v)$
   - **(b).** $h(u.v) = h(u) * h(v)$
   - **(c).** $h(u * v) = h(u).h(v)$
   - **(d).** $h(u * v) = h(u) + h(v)$

UNIVERSITY OF THE PUNJAB

Fourth Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Linear Algebra
Course Code: MATH-224 /

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Section-II

Marks=20

1. S is the set of all 2×2 matrices of the form

$$A = \begin{pmatrix} w & x \\ y & z \end{pmatrix}, \quad \text{where} \quad wz - xy = 1.$$

Show that S is a group under matrix multiplication.

2. If A and B are diagonal, show that A and B commute.

3. Consider

$$\begin{aligned} A\,|\mathbf{f}_n\rangle &= \lambda_n\,|\mathbf{g}_n\rangle, \\ \tilde{A}\,|\mathbf{g}_n\rangle &= \lambda_n\,|\mathbf{f}_n\rangle, \quad \text{with } A \text{ real.} \end{aligned}$$

(i) Prove that $|\mathbf{f}_n\rangle$ is an eigenvector of $\left(\tilde{A}A\right)$ with eigenvalue λ_n^2.(ii) Prove that $|\mathbf{g}_n\rangle$ is an eigenvector of $\left(A\tilde{A}\right)$ with eigenvalue λ_n^2.

4. Assume that the vector space R^3 has Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors

$$\mathbf{u}_1 = (1,1,1), \mathbf{u}_2 = (0,1,1), \mathbf{u}_3 = (0,0,1)$$

into an orthonormal basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

5. If $\mathbf{v}_1 = (2,-1,0,3)$, $\mathbf{v}_2 = (1,2,5,-1)$ and $\mathbf{v}_3 = (7,-1,5,8)$, the show that the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.

Section-III

Marks=30

1. Given a normal matrix A with eigenvalues λ_j, show that $A^\dagger$ has eigenvalues λ_j^*, its real part $(A + A^\dagger)/2$ has eigenvalues $\Re e(\lambda_j)$, and its imaginary part $(A - A^\dagger)/2$ has eigenvalues $\Im m(\lambda_j)$.

2. A has eigenvalues 1 and -1 and corresponding eigenvectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Construct A.

3. Solve the following system of equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 9, \\ 2x_1 - x_2 + x_3 &= 8, \\ 3x_1 + 0x_2 - x_3 &= 3. \end{aligned}$$

4. Prove that the intersection of two subgroups H and K of a group G is a subgroup.

5. Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$