

MATHEMATICS (Objective)

Time: 30 Minutes

Marks: 20

Q.No.1 You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero marks in that question. Attempt as many questions as given in objective type question paper and leave others blank. Write the letter A, B, C or D in the column (write correct option) against each question. If there is a contradiction in the bubble and hand written answer, bubble option will be considered correct.

S.#	Questions	A	B	C	D
1	Limit $\frac{\sin 5x}{x}$ is equal to:	1	Zero	$\frac{1}{5}$	5
2	If $f(x) = \dots$, then function is said to be even function:	$-f(x)$	$f(-x)$	$f(x)$	None of these
3	$\frac{d}{dx}(\log_5 x)$ is:	$5\ln x$	1	$25\log_5 x$	$\frac{1}{x\ln 5}$
4	Limit $\frac{f(x+\delta x) - f(x)}{\delta x}$	$f'(x)$	$f'(a)$	$f'(2)$	$f'(0)$
5	$\frac{d}{dx}(\sec^{-1} x) =$	$\frac{1}{x\sqrt{x^2-1}}$	$\frac{-1}{x\sqrt{x^2-1}}$	$\frac{1}{1+x^2}$	$\cot^{-1} x$
6	$\frac{d}{dx}(a^x)$	$\frac{a^x}{\ln a}$	$\frac{\ln a}{a^x}$	$a^x \ln a$	a^x
7	Maclaurins expansion of $\ln(1+x) =$	A $x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$	B $1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$	C $-x - \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$	D $x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$
8	$\int_{-\pi}^{\pi} \sin x \, dx = :$	1	2	0	-1
9	$\int \frac{e^{\tan^{-1} x}}{1+x^2} \, dx = :$	$e^{\sec x} + c$	$e^{\cot^{-1} x} + c$	$e^{\tan x} + c$	$e^{\tan^{-1} x} + c$
10	$\int_a^b f(x) \, dx = :$	$-\int_a^b f(x) \, dx$	$-\int_b^a f(x) \, dx$	$\int_{-b}^{-a} f(x) \, dx$	$-\int_{-a}^{-b} f(x) \, dx$
11	$\int \frac{dx}{x^2 + a^2} = :$	A $\ln(x + \sqrt{x^2 + a^2}) + c$	B $\frac{1}{\ln(x + \sqrt{x^2 - a^2})} + c$	C $\ln(x + \sqrt{x^2 - a^2}) + c$	D $\frac{1}{a} \tan^{-1} \frac{x}{a} + c$
12	If $\phi'(x) = f(x)$, then $\phi(x)$ is called _____ of $f(x)$:	Derivative	Integral	Differential coefficient	Area
13	$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx = :$	$\frac{1}{2}x^2 - 2x + c$	$\frac{1}{2}x^2 - \ln x + c$	$x^2 - x + \ln x + c$	$\frac{x^2}{2} - 2x + \ln x + c$
14	If P(x, y) is point in coordinate system, then x is called:	Coordinates	Ordinate	Abcissa	Origin
15	The centroid of a triangle divides each median in the ratio:	2 : 1	3 : 1	3 : 2	3 : 4
16	$x = -1$ is the solution of the inequality:	$2x + 3 \leq 0$	$2x + 3 > 0$	$x - 2 > 0$	$2x + 1 > 0$
17	The radius of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:	$\sqrt{g^2 + f^2 - c}$	$g^2 + f^2 - c$	$g^2 - f^2 + c$	$g + f^2 - c$
18	Which one is equation of point circle?	$x^2 - y^2 = 7$	$x^2 + y^2 = 4$	$x^2 + y^2 = 0$	$x^2 + y^2 = -1$
19	If the vectors $2\hat{i} + 4\hat{j} - 7\hat{k}$ & $2\hat{i} + 6\hat{j} + x\hat{k}$ are perpendicular then $x =$ _____.	8	2	1	4
20	The work done by force $\underline{F}$ through displacement $\underline{d}$ is:	$ F d \tan\theta$	$F d \sec\theta$	$F d \sin\theta$	$\underline{F} \cdot \underline{d}$

MATHEMATICS (Subjective)

Time: 02:30 Hours Marks: 80

SECTION - I

2. Write short answers of any EIGHT parts:

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- (i) Prove the identity: $\cosh^2(x) - \sinh^2(x) = 1$
- (ii) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x}{x}$.
- (iii) If $y = x^4 + 2x^2 + 2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$
- (iv) Find $\frac{dy}{dx}$ if $xy + y^2 = 2$
- (v) Find $\frac{dy}{dx}$ if $y = \sin(2x)$ at $x = \frac{\pi}{2}$
- (vi) If $\tan y(1 + \tan x) = 1 - \tan x$, show that $\frac{dy}{dx} = -1$
- (vii) Differentiate $\cot^{-1} \frac{x}{a}$ with respect to x .
- (viii) Find $\frac{dy}{dx}$ if $y = \frac{x}{\ln x}$
- (ix) Find $f'(x)$ if $f(x) = (x+1)^x$
- (x) Find stationary points for the function $f(x) = 5x^2 - 6x + 2$
- (xi) Define increasing and decreasing function.
- (xii) Find y_2 when $y = (2x+5)^{\frac{3}{2}}$

3. Write short answers of any EIGHT parts:

16

- (i) Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ if $xy - \ln x = c$
- (ii) Evaluate $\int (\sqrt{x} + 1)^2 dx$
- (iii) Evaluate $\int \frac{x^2}{4+x^2} dx$
- (iv) Find $\int \frac{\cos x}{\sin x \ln \sin x} dx$
- (v) Find $\int \ln x dx$
- (vi) Find $\int e^{-x}(\cos x - \sin x) dx$
- (vii) Find $\int \tan^4 x dx$
- (viii) Find $\int \frac{2}{x^2 - a^2} dx$
- (ix) Find $\int_1^2 (x^2 + 1) dx$
- (x) Find area bounded by $y = \cos x$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
- (xi) Graph the solution region of linear inequality $3x - 2y \geq 6$ in xy -plane.
- (xii) Define an objective function and optimal solution.

4. Write short answers of any NINE parts:

18

- (i) Find the coordinates of the point which divides internally the join of $A(-6, 3)$ and $B(5, -2)$ in the ratio $2 : 3$.
- (ii) Find h such that point $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.
- (iii) Convert the equation of straight line $2x - 4y + 11 = 0$ into two intercepts form.
- (iv) Find the point of intersection of the lines $x + 4y - 12 = 0$ and $x - 3y + 3 = 0$
- (v) Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$

(Continued P/2)

- (vi) Find the center and radius of the circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- (vii) Find equation of tangent to the circle $x^2 + y^2 = 25$ at $P(3, 4)$.
- (viii) Find focus and directrix of the parabola $x^2 = -16y$
- (ix) Find eccentricity of the ellipse $25x^2 + 9y^2 = 225$
- (x) If O is the origin and $\vec{OP} = \vec{AB}$, find point P when A and B are $(-3, 7)$ and $(1, 0)$ respectively.
- (xi) Find α , so that $|\alpha\hat{i} + (\alpha+1)\hat{j} + 2\hat{k}| = 3$
- (xii) Find scalar λ so that the vectors $2\hat{i} + \lambda\hat{j} + 5\hat{k}$ and $3\hat{i} + \hat{j} + \lambda\hat{k}$ are perpendicular.
- (xiii) Prove that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = 0$

SECTION - II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Prove that $\lim_{x \rightarrow 0} \frac{ax-1}{x} = \log_e a$ 05
- (b) If $x = a\cos^3\theta$, $y = b\sin^3\theta$, show that $a \frac{dy}{dx} + b \tan\theta = 0$ 05
6. (a) Evaluate $\int x \cdot \sin^{-1}x \, dx$ 05
- (b) Find the distance between the parallel lines $12x + 5y - 6 = 0$ and $12x + 5y + 13 = 0$. Also find the equation of the parallel line lying midway between them. 05
7. (a) Solve the differential equation $\frac{dy}{dx} = \frac{3x^2}{4} + x - 3$ if $y = 0$ when $x = 2$ 05
- (b) Minimize $z = 3x + y$ subject to the constraints $3x + 5y \geq 15$, $x + 6y \geq 9$, $x \geq 0$, $y \geq 0$ 05
8. (a) Find equation of circle passing through $A(3, -1)$, $B(0, 1)$ and having center at $4x - 3y - 3 = 0$ 05
- (b) Prove that the line segments joining the mid points of the sides of a quadrilateral taken in order form a parallelogram using vectors. 05
9. (a) Find the centre, foci, eccentricity, vertices of hyperbola $9x^2 + 12x - y^2 - 2y + 2 = 0$ 05
- (b) Find the volume of tetrahedron with the vertices $A(2, 1, 8)$, $B(3, 2)$, $C(2, 1, 4)$ and $D(3, 3, 0)$ 05