

FEDERAL PUBLIC SERVICE COMMISSION



COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100
NOTE: (i) Attempt FIVE questions in all by selecting THREE questions from SECTION – A and TWO questions from SECTION – B . All questions carry equal marks. (ii) Use of Scientific Calculator is allowed. (iii) Extra attempt of any question or any part of the attempted question will not be considered.	

SECTION - A

- Q.1.** (a) Prove that both the order and index of a subgroup of a finite group divide the order of the group. (10)
- (b) Define cyclic group. Also prove that every cyclic group is abelian. (05)
- (c) Define order of a permutation in S_n . Find the order of $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ (05)
- Q.2.** (a) Let ϕ be a homomorphism of a group G onto another group H with Kernel K. Prove that G/K is isomorphic to H. (10)
- (b) Show that the vectors (3, 0, -3), (-1, 1, 2), (4, 2, -2) and (2, 1, 1) are linearly dependent over R. (10)
- Q.3.** (a) Define the dimension of a vector space V over a field F. Also prove that all basis of a finite dimensional vector space contain the same number of elements. (10)
- (b) A linear transformation $T : U \rightarrow V$ is one-to-one iff $N(T) = \{0\}$. (10)
- Q.4.** (a) Examine the following system for a non-trivial solution: (10)
- $$\begin{aligned} x_1 - x_2 + 2x_3 + x_4 &= 0 \\ 3x_1 + 2x_2 + x_4 &= 0 \\ 4x_1 + x_2 + 2x_3 + 2x_4 &= 0 \end{aligned}$$
- (b) Show that $\bar{Z}_3 = \{\bar{0}, \bar{1}, \bar{2}\}$ form finite field with addition and multiplication of residue classes modulo P. (10)
- Q.5.** (a) Let V be a vector space of n – square matrices over a field R. Let U and W be the subspaces of symmetric and anti symmetric matrices respectively. Then show that $V = U \cup W$. (10)
- (b) Let A and B be matrices of order 6 such that $\det(AB^2) = 72$ and $\det(A^2B^2) = 144$. Find $\det(A)$ and $\det(AB^6)$ (10)

SECTION – B

- Q.6.** (a) Sketch the curve $r^2 = a^2 \cos 2\theta$, $a > 0$. (10)
- (b) Find the tangent and the normal to the circle $x = a \cos \theta$, $y = a \sin \theta$ at the point P ($a \cos \alpha$, $a \sin \alpha$). (10)
- Q.7.** (a) Find the Pedal equation of the parabola $y^2 = 4a(x + a)$ (10)
- (b) Find the equations for a straight line passing through the points $P_1(x_1, y_1, z_1), P_2(x_2, y_2, z_2)$. Find the co-ordinates of the point where this line cuts the yz-plane. (10)
- Q.8.** (a) Determine the curvature of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ at the point (x,y). (10)
- (b) Find the equation of the plane which passes through the point (3, 4, 5) has an x – intercept equal to -5 and is perpendicular to the plane $2x + 3y - z = 8$. (10)
